Stochastic Navigation of Unmanned Aerial Vehicles (UAVs) for Border Patrolling: A Response Surface Methodology Approach

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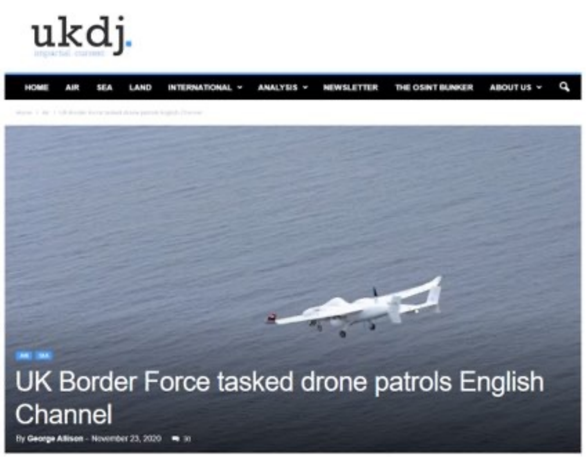
1. Introduction: UAVs in Border Patrolling

The predominant strategy for border patrol operations is deploying **human** resources and **manned** ground vehicles.

However, this approach is frequently costly, occasionally ineffective, and can even pose **risks** to the individuals involved.

A better approach : using **UAVs**



EU and UK are already employing UAVs for border patrol purposes.

"monitor and protect physical crossings of people and goods into and out of the UK's territory"[1]

2. Problem

the draft '2025 UK Border Strategy' [1]

- Detect and reduce threats as far as possible before they reach the border to ensure effective interventions and enforcement of controls at the right point in the journey
- The border will be "highly digitised and **automated**"

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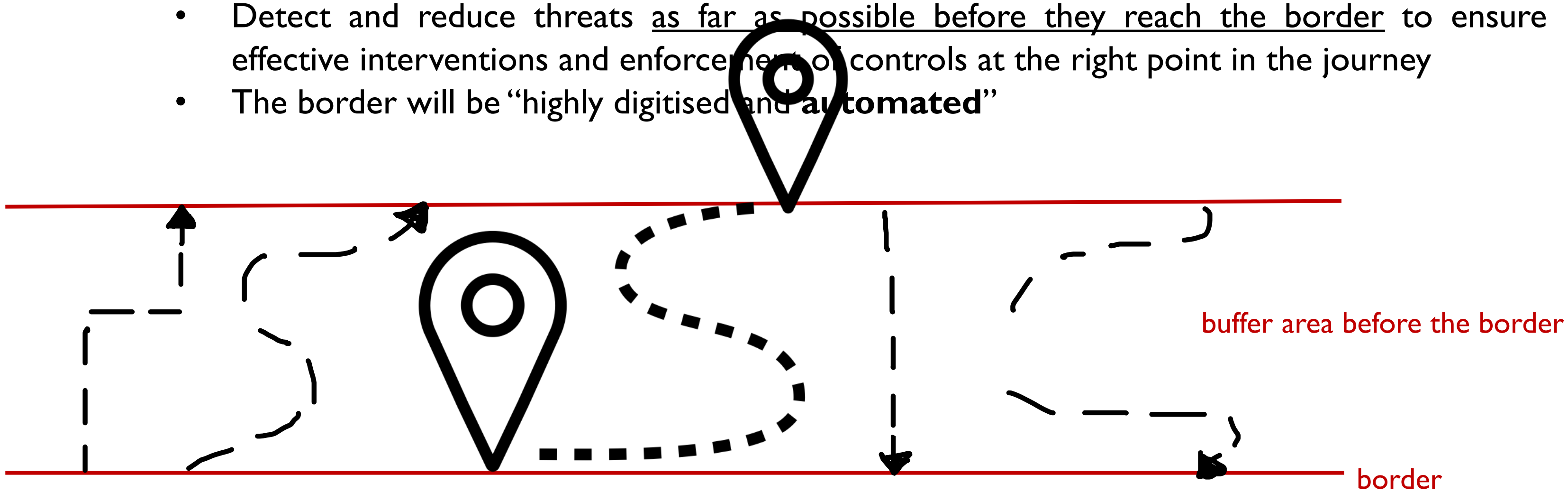

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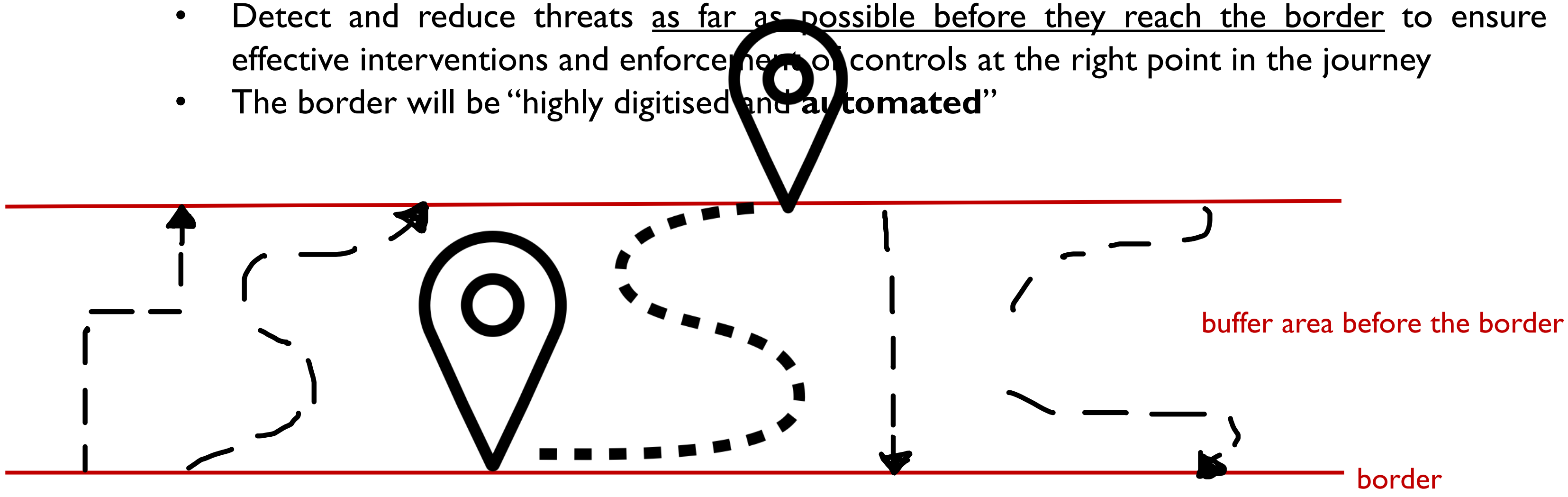

→The threats try to pass through the border in both ways in sequential order with a stochastic pattern to avoid getting detected. If they followed a deterministic pattern, UAVs would easily learn it and detect each and every threat.

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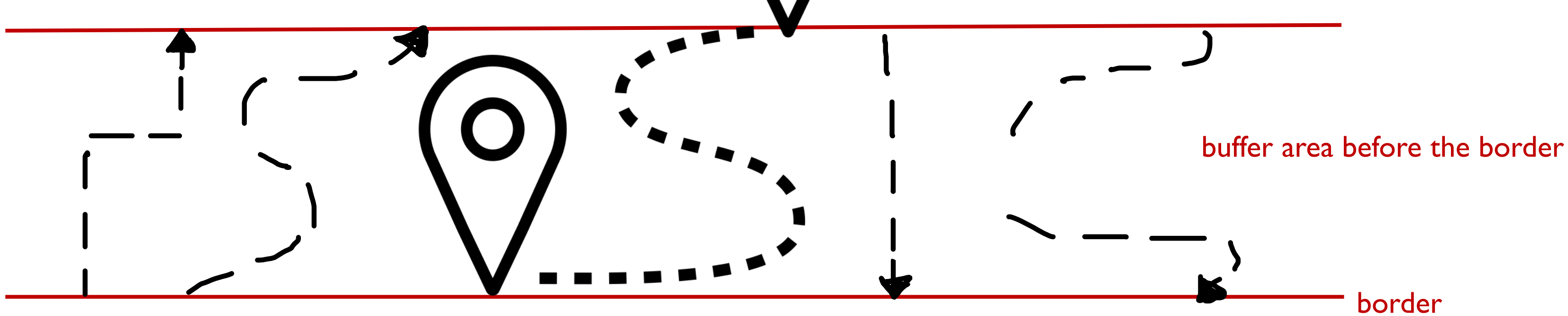

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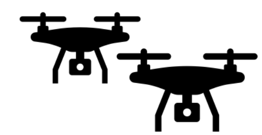

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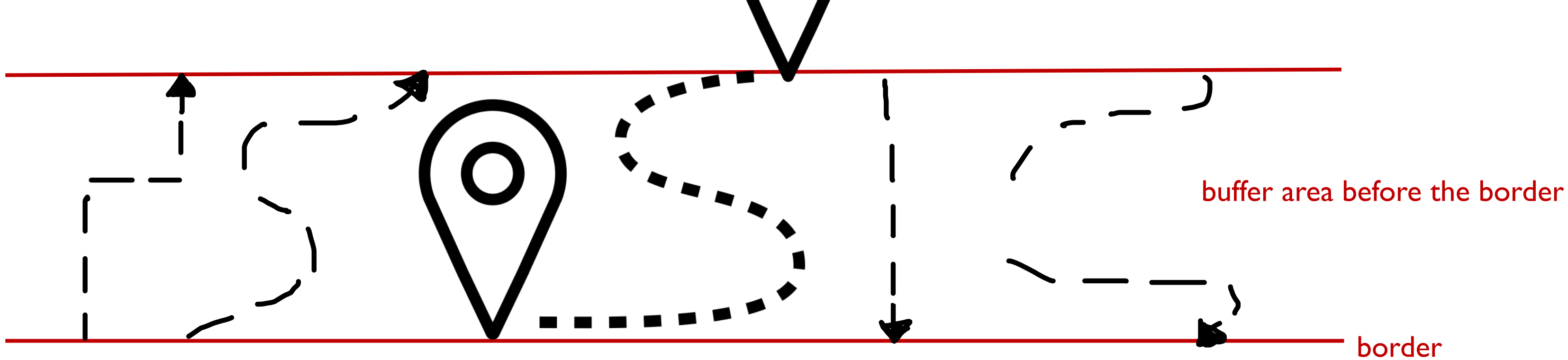

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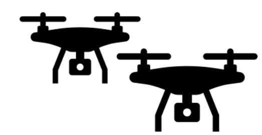

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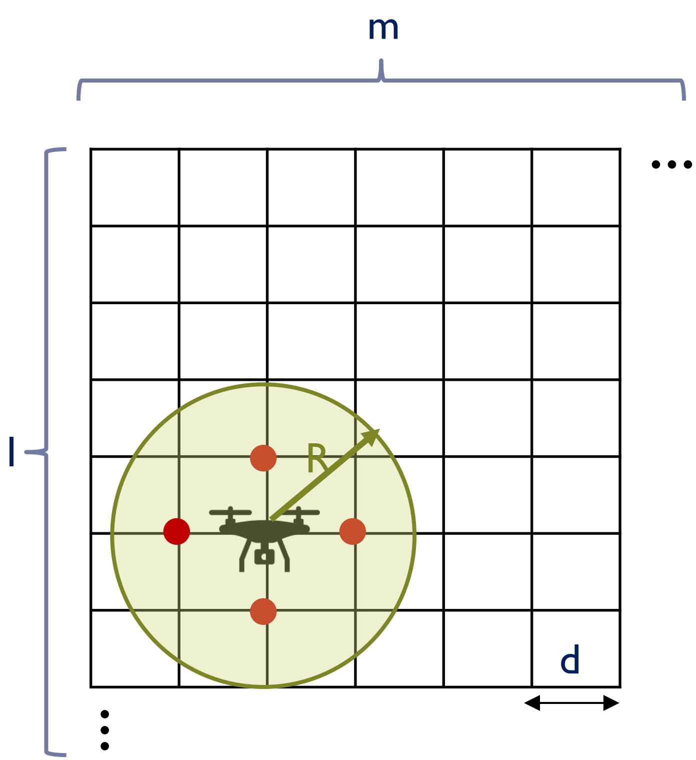
<u>Our aim</u>

Finding a **stochastic strategy** for the UAV to detect the threats under uncertainty **before** they reach the border

3. Modelling the Problem: Creating the Environment



Assumptions

- UAV moves on a two-dimensional lattice graph which can be defined as a P_m x P_l in the graph Cartesian product as paths with m and l edges (and hence m+1 and l+1 vertices), respectively.
- UAVs can only make 90° movements through four directions with probability distribution $\boldsymbol{P=[\, p_W, p_E, p_S, p_N \,]}$.
- UAVs move stepwise and can only make one step in a time step.
- UAVs can only make a limited number of moves, denoted by *L*. It is the distance limit on UAVs.
- UAV's detection range is accepted as a radius R projected onto the mission area. If the targets fall within this range can be detected by the UAV.

3. Modelling the Problem: Objective & Model

Objective is to **identify the probabilities** in such a way we **maximise** the **chance** to detect the threat

↓ stochasticity from UAV & threat movement

simulation optimisation problem
S number of simulations

↓ noise, ε

u^t: location of the UAV,(u_1, u_2), at time step $t, u_1 \in [0, m+1], u_2 \in [0, l+1], 0 \leq t \leq L$

v^t: location of the target, (v_1, v_2), at time step $t, v_1 \in [0, m], v_2 \in [0, l], 0 \leq t \leq L$

UAV moves with a function *F(P, ε)*

$$T = \begin{cases} \text{t}, & \text{if } \|\text{u}^t, v^t\| < R, \quad 0 < \text{t} \leq L \\ M, & \text{otherwise} \end{cases} \qquad D_k = \begin{cases} 1, & \text{if } 0 < t \leq L \\ 0, & \text{if } t = M \end{cases}$$

$$\text{maximise} \qquad \frac{1}{S} \sum_{k=1}^{S} D_k$$

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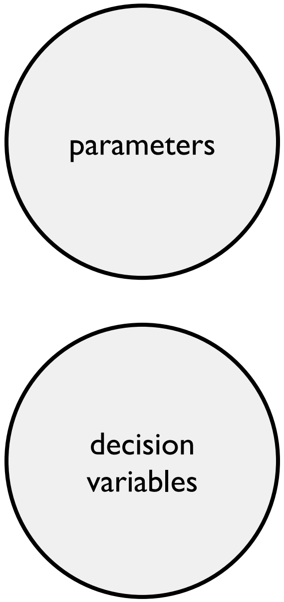
→ **PoS**

4. Methodology: Simulation Approximation Approach



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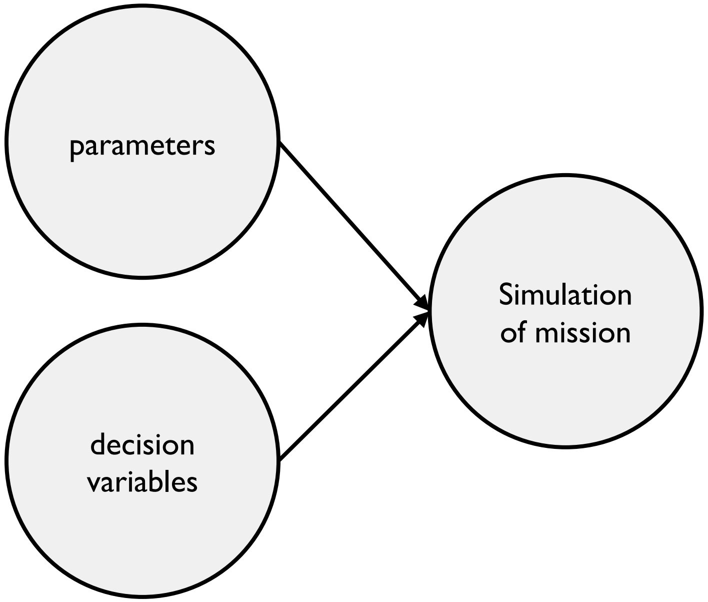

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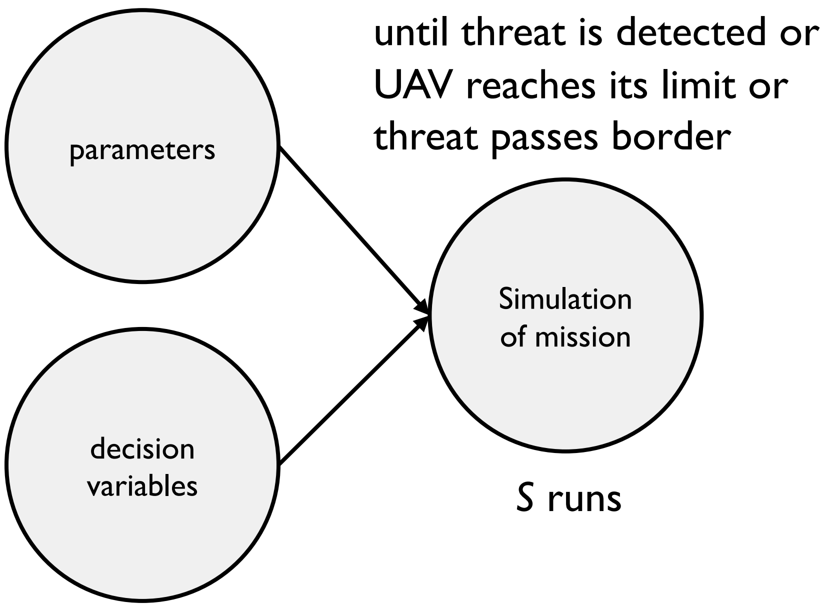

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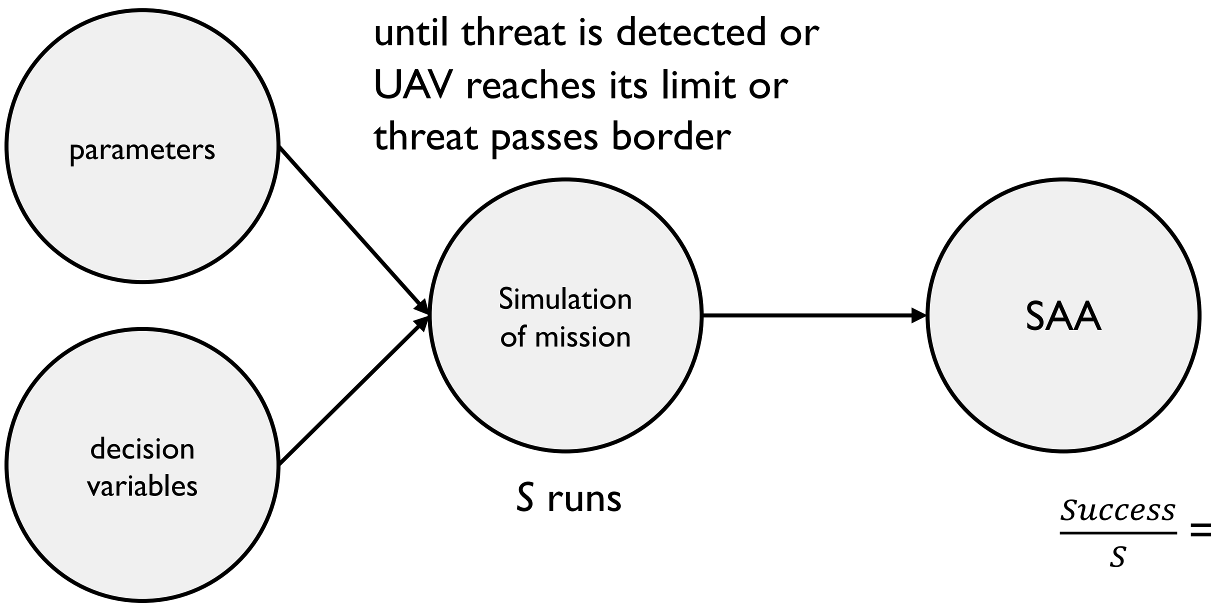

$$\frac{Success}{S} = PoS$$

$$\frac{Step}{Success} = E[T]$$

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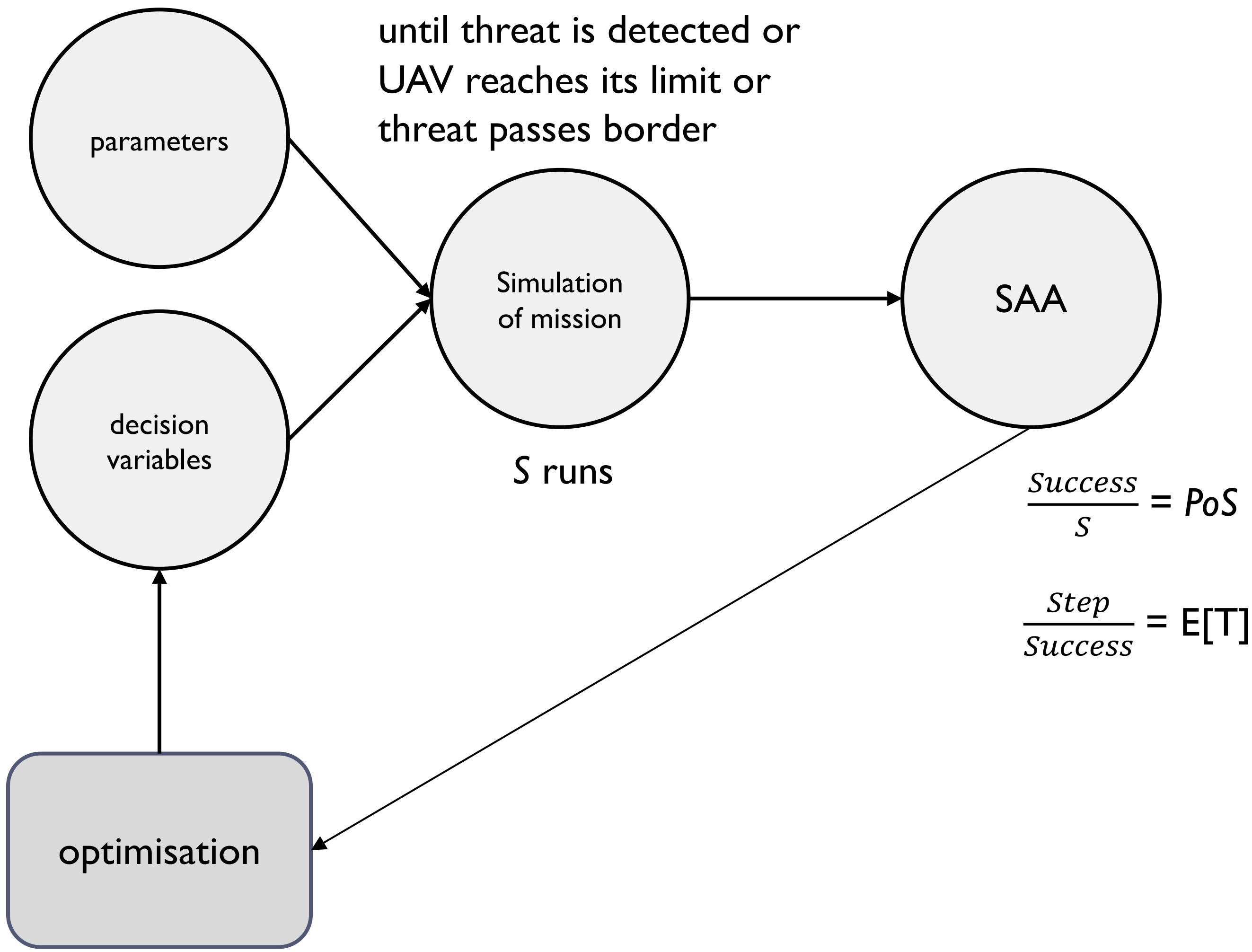

4. Methodology: Optimisation of *probability set, P*

- Simulated Annealing Algorithm
- Stochastic Nelder-Mead Method
- Response Surface Methodology with Radial Basis Function

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4. Methodology: Optimisation of *probability set, P*

- Simulated Annealing Algorithm ⊗ worse execution time
  worse results
- Stochastic Nelder-Mead Method ⊗ worse execution time
  similar results

- Response Surface Methodology with Radial Basis Function

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- Simulated Annealing Algorithm ⊗ worse execution time
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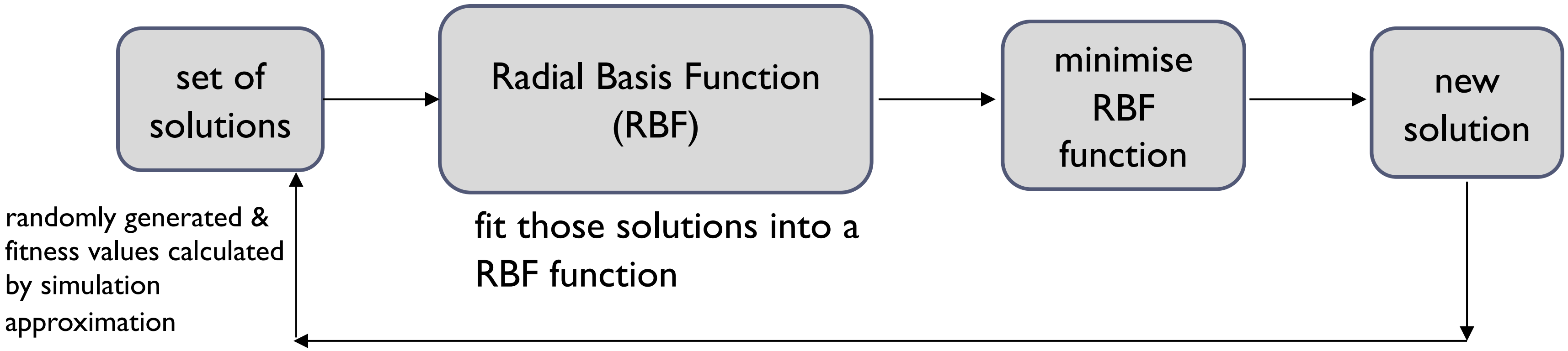
- Stochastic Nelder-Mead Method ⊗ worse execution time
  similar results
- Response Surface Methodology with Radial Basis Function ✓

A viable method for a complex system entails a response $y = F(x)$ that relies on the parametric design variables $x = [x_1, \ldots, x_n]^T$, which in our context refer to $P = [p_W, p_E, p_S, p_N]$. An appropriate estimation of the function F(x), F(P) in our case, will be formulated, given that the function itself is assumed to be unknown and complicated.

4. Methodology: Radial Basis Function



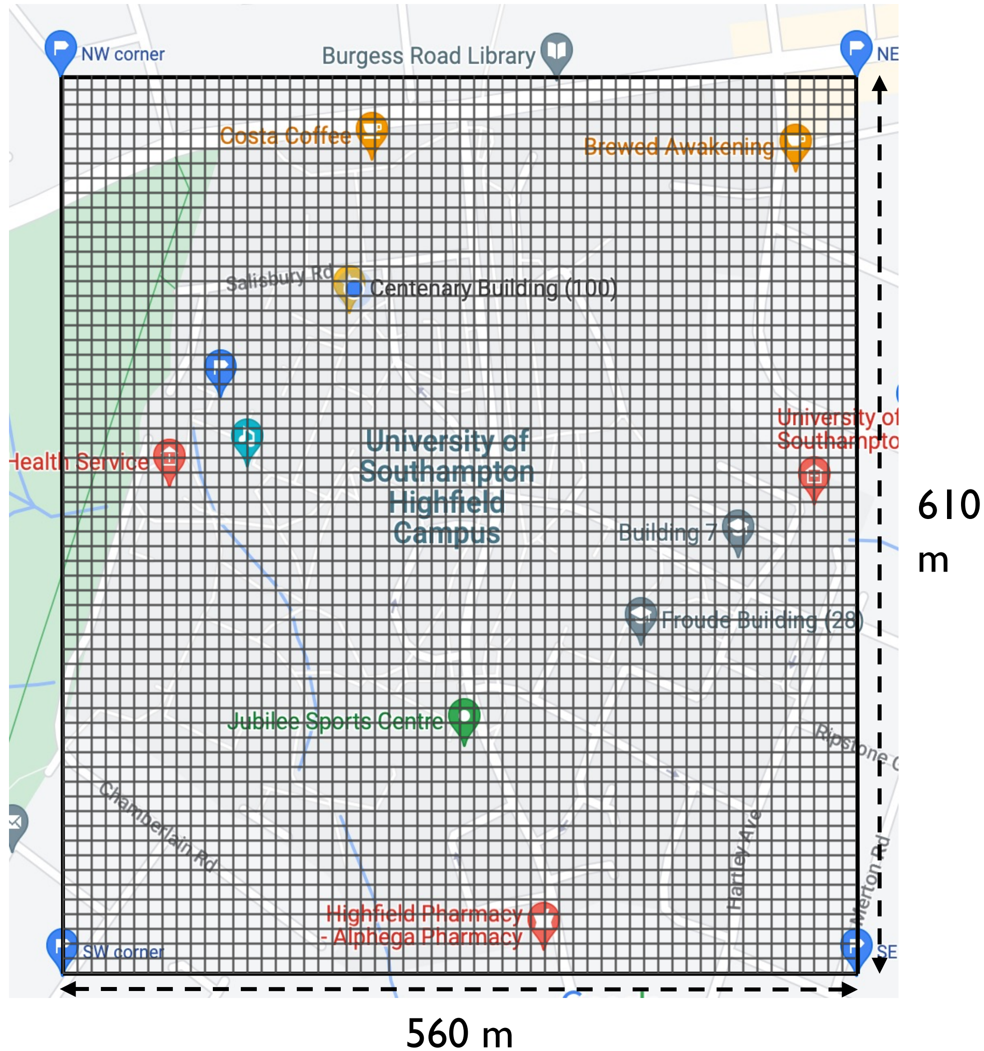
Radial basis functions have the form

$$F(\mathbf{x}) = \sum_{i=1}^{m} \beta_i \phi(r_i, c), \qquad (1)$$

where $r_i(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_i\|$ is the distance of the point $\mathbf{x}$ from the ith data point $\mathbf{x}_i$ in the parameter space, $\|\cdot\|$ denotes the Euclidean norm, $\phi(\cdot)$ is a suitably chosen radial basis function, c is a user-defined constant which is usually required to be non-negative, and β_i is the radial basis coefficient corresponding to the ith data point.

5. Numerical Examples

Number of UAVs	1 - 2	
Number of decision stages to change *P*	1 - 2	
Mission area	University of Southampton Highfield Campus	
	length	width
	610 m	560 m
How much faster UAVs then threats?	10 times	
d	10 m	
R	20 m	
Number of paths that threat can choose	differs (1- 3- 6)	
number of starting solutions	20	



5. Numerical Examples

Scenario 1	Scenario 2	Scenario 3	Scenario 4
1 UAV moves with a set of probabilities decided at the beginning of the mission.	1 UAV makes n moves with a probability set decided at the beginning of the mission and uses another set of probabilities after n moves for (L-n) moves.	2 UAVs move with a set of probabilities decided at the beginning of the mission. The mission is accepted as successful if one of them detects the target.	2 UAVs, similar to Scenario 3, except they start from 2 different corners: South-West and North-East
$[p_W, p_E, p_S, p_N]$	$[p_{W1}, p_{E1}, p_{S1}, p_{N1}, p_{W2}, p_{E2}, p_{S2}, p_{N2}]$	$[p_{W1}, p_{E1}, p_{S1}, p_{N1}, p_{W2}, p_{E2}, p_{S2}, p_{N2}]$	$[p_{W1}, p_{E1}, p_{S1}, p_{N1}, p_{W2}, p_{E2}, p_{S2}, p_{N2}]$
1 decision stage 1 UAV 4 decision variables	2 decision stages 1 UAV 8 decision variables	1 decision stage 2 UAVs 8 decision variables	1 decision stage 2 UAVs 8 decision variables

+ with ground sensor

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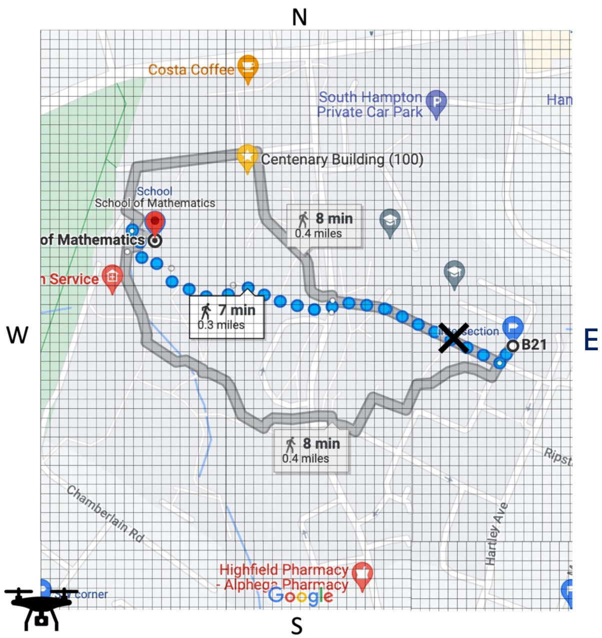
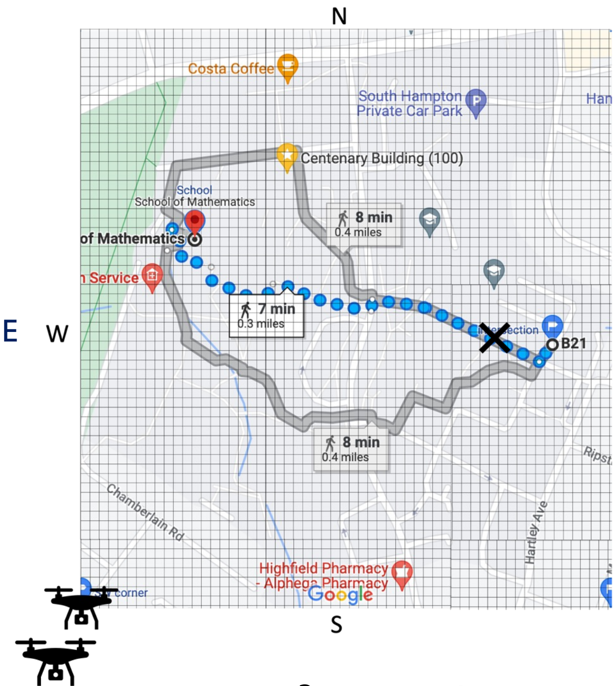
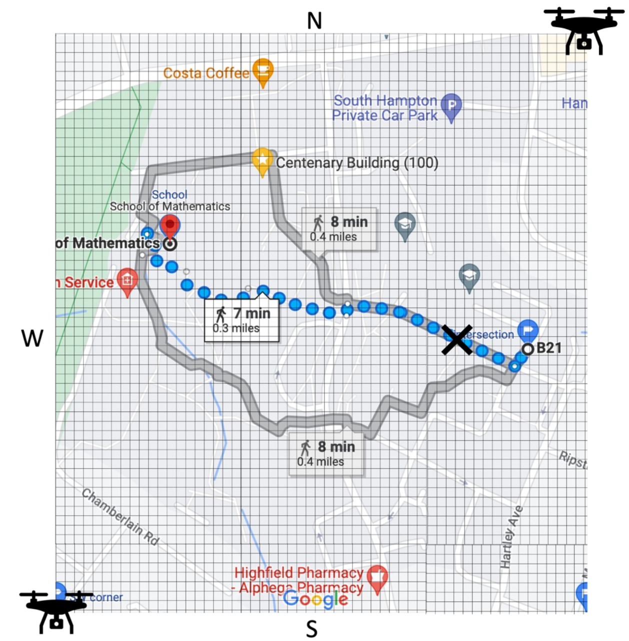


scenario 1
scenario 2 + ground sensor



scenario 3 + ground sensor



scenario 4 + ground sensor

6. Results

Scenario	best solution (W, E, S, N)	PoS	E[T]	standard deviation	execution time (sec)
1) 1 decision stage 1 UAV 4 dv	[0.114, 0.252, 0.251, 0.383]	0.231	212.94	36.42	167.99
1) + sensor	[0.676, 0.001, 0.001, 0.322]	0.414	88.38	1.59	117.05
2) 2 decision stages 1 UAV 8 dv	[0.299, 0.201, 0.042, 0.458] [0.134, 0.632, 0.056, 0.178]	0.283	122.13	10.85	173.81
2) + sensor	[0.001, 0.698, 0.001, 0.300] [0.659, 0.001, 0.001, 0.339]	0.422	66.73	1.39	126.52
3) 1 decision stage 2 UAVs, same corner 8 dv	[0.199, 0.594, 0.001, 0.206] [0.001, 0.603, 0.001, 0.395]	0.457	82.16	19.67	304.94
3) + sensor	[0.030, 0.629, 0.001, 0.340] [0.708, 0.001, 0.001, 0.290]	0.705	85.1	5.42	178.35
4) 1 decision stage 2 UAVs, different corners 8 decision variables	[0.001, 0.001, 0.791, 0.207] [0.201, 0.001, 0.797, 0.001]	0.506	43.2	1.28	276.12
4) + sensor	[0.010, 0.454, 0.135, 0.401] [0.697, 0.001, 0.001, 0.301]	0.685	76.17	8.45	208.6

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information effect
(sensor)

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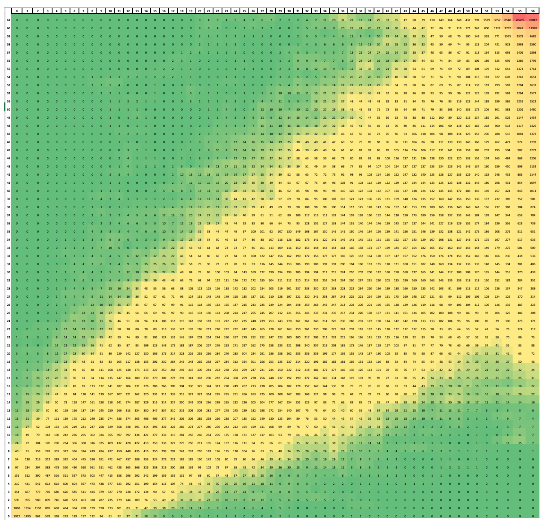
one more decision stage effect

one more UAV effect

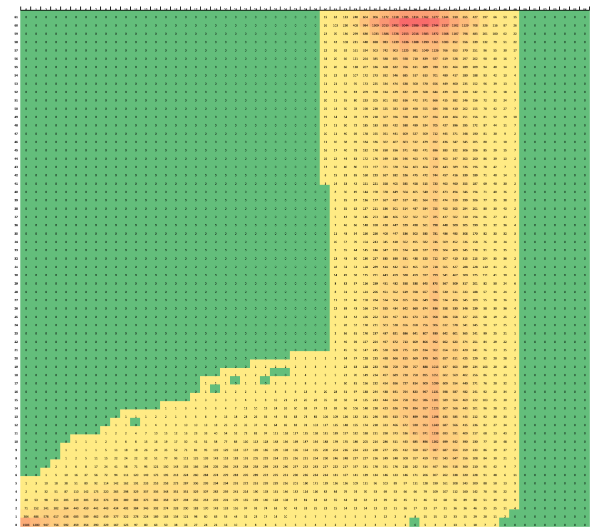
different starting points effect

Heatmaps of frequency of UAV visits over the simulation in the mission area

scenario 1



scenario 2

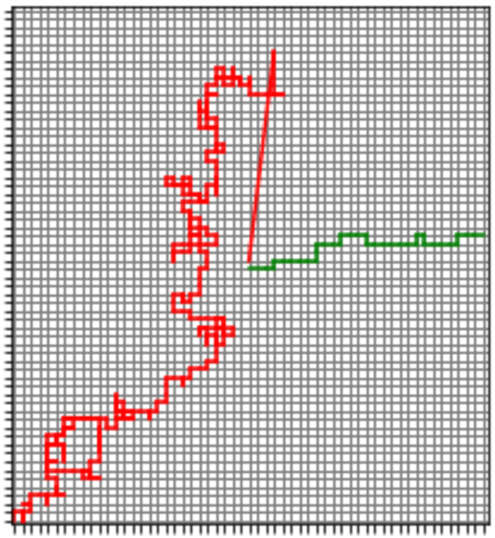


1 decision stage
1 UAV
4 dv

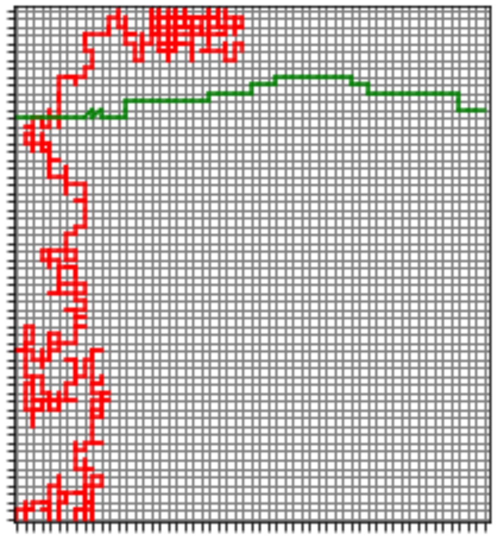
2 decision stages
1 UAV
8 dv

6. Experiment: random walk for the target

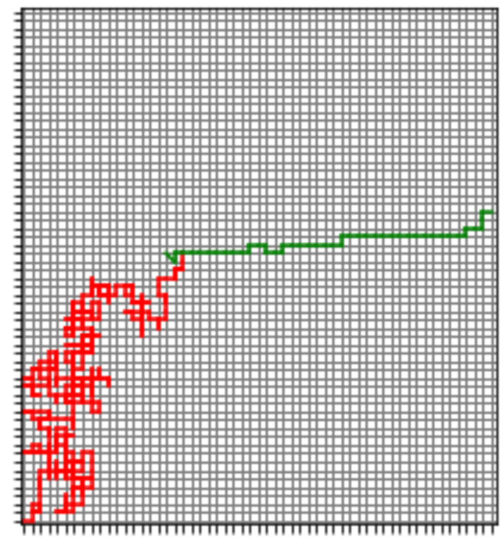
Target moves with probability set P = [0.8, 0.1, 0.1, 0.1] (W, E, S, N)



detected with signal received



couldn't detect, no signal

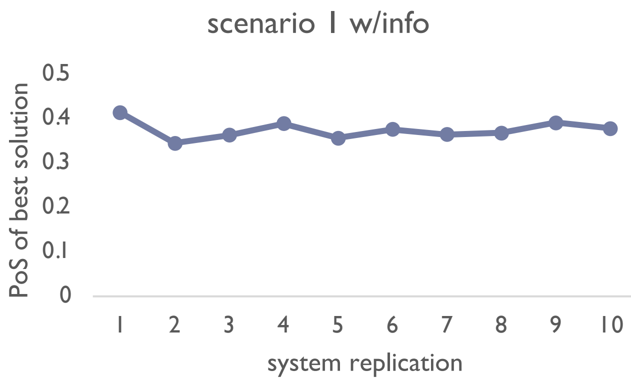


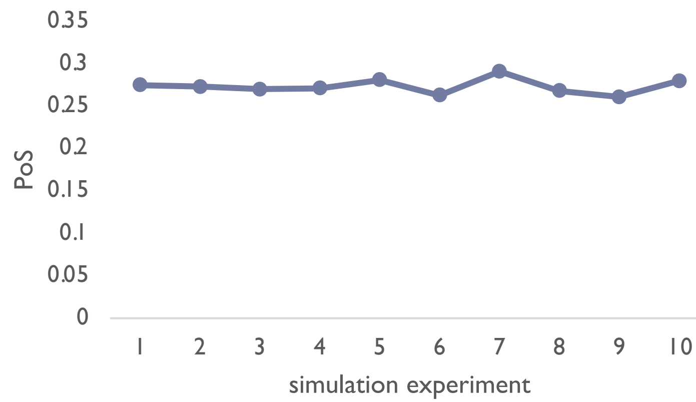
detected without signal

best solution (W, E, S, N)	PoS	ET	SD	execution time (secs)
[0.172, 0.209, 0.264, 0.355] [0.321, 0.089, 0.161, 0.429]	0.205	397.99	68.99	263.92

7. Challenges

- NOISE

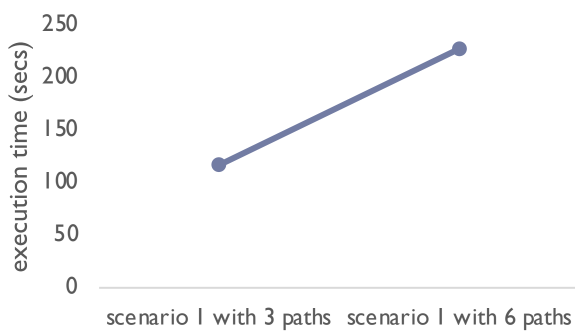


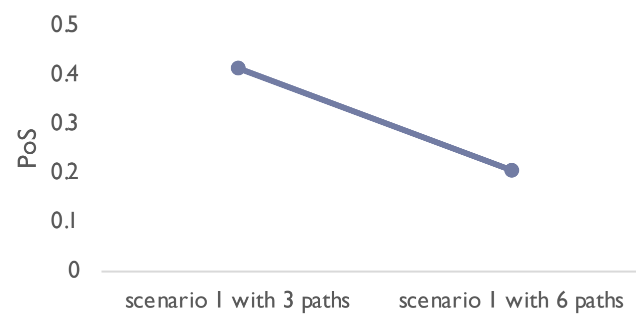


- Execution Time

more decision stages/ → more decision variables → increased execution time
more UAVs

Ex.





9. Further Study

- Decreasing the noise
- More decision stages for UAVs
- Communication of UAVs
- Application of the model in different operations: monitoring wildlife, disaster relief operations, search and rescue operations

Thanks!

Happy to answer any questions!

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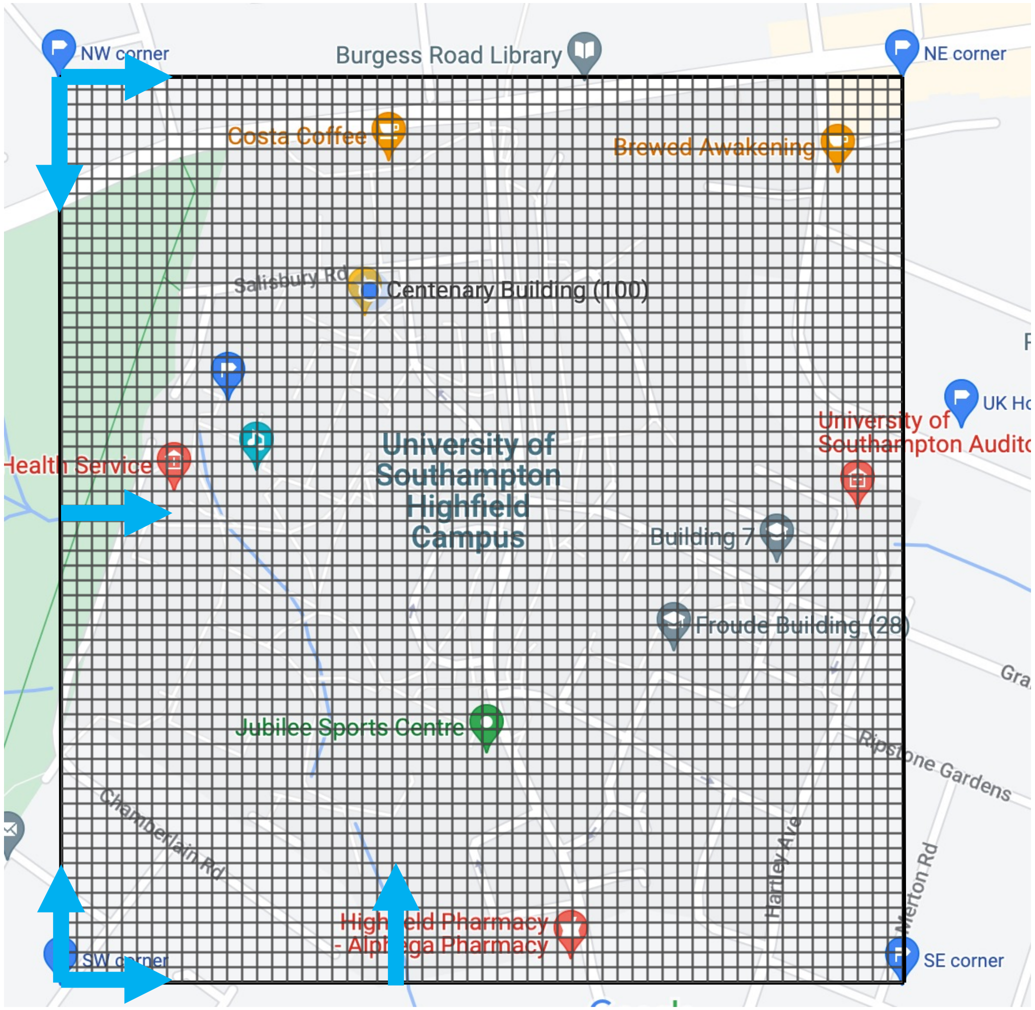
References

1 Peter Burt and Jo Frew: 'Crossing A Line | The use of drones to control border'. Drone Wars UK, December 2020. https://dronewars.net/wp-content/uploads/2020/12/DW-Crossing-a-Line-WEB.pdf

How UAV moves in the border?



Mathematical Model

$$T = \begin{cases} \mathrm{t}, & \text{if } \|\mathrm{u}^t, v^t\| < R, \quad 0 < \mathrm{t} \leq L \\ M, & \text{otherwise} \end{cases}$$

$$\text{minimise} \quad E[T] \simeq \frac{1}{S} \sum_{k=1}^{S} Y(P, \xi_k)$$

s.t.

$$\sum_i p_i = 1 \quad i \in W, E, S, N$$

$$0 < p_i < 1 \quad i \in W, E, S, N$$

Steps of RBF

1. A particular basis function $\phi(r, c)$ is chosen. In this paper we will be using functions from the five classes in (14). Within each class, a particular member is determined by the value assigned to the constant c.
2. The data points are scaled so each component of $\mathbf{x}$ is in the range $-1 \leqslant x_i \leqslant 1$.
3. An $m \times m$ symmetric matrix $\mathbf{R} = [r_{ij}]$ is constructed, where m is the number of data points. Each entry r_{ij} of this matrix is the Euclidean distance between data points $\mathbf{x}_i$ and $\mathbf{x}_j$.
4. The chosen basis function $\phi(r, c)$ is applied component-wise to the matrix $\mathbf{R}$, creating the matrix $\mathbf{A}$.
5. The matrix equation $\mathbf{A}\boldsymbol{\beta} = \mathbf{F}$ is solved, where each component F_i of the vector $\mathbf{F} = [F_1, \ldots, F_m]^\top$ is the objective function value at the corresponding data point $\mathbf{x}_i$. The resulting vector $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_m]^\top$ is the vector of radial basis coefficients.
6. The model function value $f(\mathbf{x})$ at an arbitrary point $\mathbf{x}$ within the parameter space is found in the following manner. A vector $\mathbf{g}(\mathbf{x}) = [g_1, \ldots, g_m]^\top$ is constructed whose components g_i are obtained by the formula $g_i = \phi\ (r_i(\mathbf{x}), c)$, where $r_i(\mathbf{x})$ is the distance between $\mathbf{x}$ and the ith data point $\mathbf{x}_i$. Then

$$f(\mathbf{x}) = \boldsymbol{\beta}^\top \mathbf{g}(\mathbf{x}) = \sum_{i=1}^{m} \{\beta_i g_i\}. \quad (15)$$

Additional Graphs