DATA ENVELOPMENT ANALYSIS REVIEWED

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ABSTRACT

Data Envelopment Analysis (DEA) is an approach to efficiency measurement that has been the subject of much academic enquiry. During the last two decades or so thousands of papers, books and dissertations have been written. However, it is essential to regularly assess the achievements of this fast growing field and identify avenues for further research. This paper reviews the state of the art of DEA. It defines the approach, illustrates its use and discusses its strengths and limitations. After discussing the basic models, it describes some of its extensions, applications and software available. The issues surrounding the practical application of DEA are reviewed and directions for further research are presented. The paper concludes by suggesting that the use of DEA is still the province of specialists and that further research should consider ways to make it user-friendlier.

KEY WORDS

Data Envelopment Analysis, Efficiency, Performance Measurement.

1. INTRODUCTION

Efficiency is often described as a managerial tool for assessing the ability of an organisation to generate outputs from a given amount of inputs. The Operational Research/Management Science (OR/MS) and Economics literature identify three main approaches to tackle the problem of efficiency measurement. The ratio approach measures efficiency by calculating simple ratios based on accounting measures. The parametric approach assumes the existence of a specific functional form (mostly, a regression model or a known production function, such as Cobb-Douglas) relating inputs to outputs. The non-parametric approach (e.g., Data Envelopment Analysis = DEA), in contrast, does not require any prior assumption on the functional form of the input-output relationship. DEA focuses on a search for extreme relationships of inputs and outputs.

In the last few years, DEA has experienced notable interest from academic researchers, as witnessed by the large volume of literature that has been produced. Seiford (1996) gave a lucid account of the field till 1995. Emrouznejad (2001) and Tavares (2002) list over 2500 papers, 50 books and 170 dissertations since 1978. It is the subject of study of many undergraduate and postgraduate courses (in particular, OR/MS courses), as well as the focus of attention of several research centres around the world (e.g., the IC^2 Institute at the University of Texas at Austin, Euro Working Group in DEA and Performance Measurement). Increasingly, the DEA methodology seems to be attracting the attention of OR practitioners. Evidence of this is perhaps the publication of papers in trade and practitioner journals such as *Fortune* (Norton, 1994), *Interfaces* (Athanassopoulos and Giokas, 2000) and *OR Insight* (Dyson, 2000). However, as will become clear, this methodology has its own flaws and, consequently, pertinent questions still remain unanswered.

The purpose of this paper is to (i) outline the principal characteristics of DEA, (ii) review some state of the art issues, (iii) and point out some unresolved questions. Given the fast growing and fragmented nature of this field, this seems to be a sensible thing to do. It is also a useful starting point for neophytes. The paper unfolds as follows. The next section provides an introduction to DEA terms and concepts. Section 3 focuses attention on two basic DEA models and their interpretation. Section 4 is devoted to presenting some key extensions and enhancements of DEA. It also discusses software available and illustrates potential uses of DEA for assessing performance. Section 5 discusses various issues that are the focus of ongoing research and, finally, Section 6 concludes the paper.

2. DEFINING DEA

The birth of DEA lies in the seminal research work undertaken by Farrell (1957) and later popularised by Charnes *et al.* (1978, 1979) (CCR), which are indisputably recognised as the principal basis of the non-parametric methodology to assess efficiency. The original CCR model was extended by Banker *et al.* (1984) (BCC) to incorporate variable returns to scale. Consequently, the CCR and BCC models are the two basic models that are usually associated with DEA.

Charnes *et al.* (1981, p. 668) defined DEA as a:

> ...mathematical programming model applied to observational data [that] provides a new way of obtaining empirical estimates of extremal relations - such as the production functions and/or efficient production possibility surfaces that are a cornerstone of modern economics. The resulting extremal relations are used to envelop the observations in order to obtain the efficiency measures...

Hence, DEA is an OR/MS methodology based on mathematical programming theory for assessing the relative efficiency of Decision Making Units (DMUs or units for short)[1], having the same multiple inputs and outputs. The objective of DEA is to discover whether a particular DMU is or not performing relatively more efficiently in comparison to the set of units under evaluation, given its observed inputs and outputs. This is achieved by identifying a relatively 'best practice' subset of DMUs that enables to construct an empirically based production possibility frontier, referred to as 'efficiency frontier' or 'envelope'. An efficiency measure is, then, defined to each DMU based on its position relative to the frontier of 'best practice'. If a certain DMU lies on the frontier then it is considered relatively efficient. Each DMU not in the frontier is deemed relatively inefficient.

Figure 1 illustrates the DEA methodology for the single input, two-output case. The line connecting all extreme DMUs and the axes is the efficiency frontier. It indicates the maximum combinations of outputs that can be obtained from a given set of inputs. Since DMU1, DMU2 and DMU3 lie on the efficiency frontier, they are efficient. DMU4 and DMU5 are not on the efficiency frontier and therefore are inefficient. The efficiency score of DMU4 can be calculated by comparing it to a composite unit (DMU'4) constructed from DMU1 and DMU2 (its reference set). The same applies to DMU5. However, since its composite unit is

[1] Following Golany and Roll (1989) the term 'Decision Making Units' means an homogeneous group of firms, departments or administrative units (non-profit or profit-oriented) that (1) perform the same tasks, (2) operate under the same set of market conditions, and (3) have identical inputs and outputs. Examples are branches of a bank, firms of the grocery industry, academic departments of a university, etc.

outperformed by DMU1, the distance between DMU'5 and DMU1 represents the 'slack' that must be overcome to render DMU'5 efficient. For useful introductions to the theory and practice of DEA see, for example, Dyson *et al.* (1990), Cooper *et al.* (1999b) and Thanassoulis (2001).

[Insert figure 1 about here]

From the above description, it follows that DEA, as its name suggests, is based on data analysis. Seiford (1996, p. 103) states the importance of this view, "DEA is now recognised as a versatile and effective tool for data analysis and is often used as an exploratory technique (E-DEA) for visualising the data". The term 'envelopment' emphasises the fact that the inefficient DMUs are enveloped by the efficient DMUs. DEA can have other uses beyond efficiency measurement, such as (Boussofiane *et al.*, 1991):

- Identifying efficient operating practices;
- Target setting;
- Monitoring efficiency changes over time;
- Resource allocation.

The main strength of DEA is that it can handle the multiple input, multiple output case without requiring any assumption of a functional form relating inputs to outputs. Another advantage is that DEA tries to overcome the difficulty of quantifying qualitative factors. This is often the situation of service and public sectors, but also private companies if some factors cannot be easily quantifiable. Moreover, DEA is not only good at comparing various DMUs but also can be used to identify a course of action for improving the performance of inefficient DMUs, i.e., the reduction in input levels or increases in output levels necessary for efficiency. Epstein and Henderson (1989) provide an in-depth discussion of the advantages and disadvantages of DEA.

DEA methodology

The application procedure of DEA comprises several stages. The first relates to defining and selecting the DMUs to be assessed. The identification of the relevant inputs and outputs to be analysed is another important stage. The final stage is the application of the DEA models and interpretation of the results. Each stage, in turn, involves several steps and raises several issues requiring careful handling. Such issues are discussed, for example, in Thanassoulis *et* *al.* (1987), Golany and Roll (1989), Boussofiane *et al.* (1991), Charnes *et al.* (1994) and Dyson *et al.* (2001).

3. The Basic DEA Models and their Interpretation

Mathematically, the DEA methodology (the primal CCR model) solves the following non-linear fractional programming problem to determine the relative efficiency of each DMU:

$$\max \qquad e_k = \sum_{r=1}^{s} u_r y_{rk} \Big/ \sum_{i=1}^{m} v_i x_{ik}$$

subject to:

$$\sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \leq 1 \quad ; \quad j = 1,2,\dots,k,\dots,n \qquad (1)$$
$$u_r, v_i \geq e \quad ; \quad r = 1,2,\dots,s; \quad i = 1,2,\dots,m$$

Where:

e_k = the relative efficiency score of DMU k (the DMU being assessed),
n = the number of DMUs being assessed,
y_{rj} = the observed amount of output r produced by DMU j,
x_{ij} = the observed amount of input i used by DMU j,
u_r, v_i = the weights given to output r and input i, respectively,
s, m = the number of output and input measures, respectively,
e = a small positive number.

This formulation can be converted, as shown by Charnes *et al.* (1978), into an ordinary linear programming model by setting the denominator in the objective function equal to a constant (usually unity) and maximising the numerator. The resultant formulation is the following:

$$\max \qquad e_k = \sum_{r=1}^{s} u_r y_{rk}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad ; \quad j = 1,2,\dots,k,\dots,n \qquad (2)$$
$$u_r, v_i \geq e \quad ; \quad r = 1,2,\dots,s; \quad i = 1,2,\dots,m$$

For a complete DEA evaluation, the above CCR model must be solved n times, each time the objective function suitably changed for the DMU being investigated. The following are some important interpretations of the previous models:

1) The relative efficiency score e_k is calculated as the ratio of the sum of the weighted outputs to the sum of its weighted inputs. The CCR model, thus, transforms the multiple

output-multiple input case into a single ratio of a single 'virtual' output to a single 'virtual' input (Seiford and Thrall, 1990).

2) The decision variables are the weights (u_r, v_i) for both outputs and inputs and they determine the importance assigned to each factor in determining the efficiency score. These weights are chosen freely so that the DMU being assessed is put in the 'best possible light', subject to the constraint that the relative efficiency score of any other DMU with these weights will not be greater than unity[2]. It should also be noted that each weight must be greater than or equal to a small positive number e (also called the non-Archimedian infinitesimal), whose purpose is to avoid any factor being totally ignored when calculating the relative efficiency (Dyson *et al.*, 1990);

3) Each DMU being assessed will either have an efficiency score $e_k^* = 1$ or $e_k^* < 1$ (the star superscript indicates its optimal value). If $e_k^* = 1$ then the DMU is relatively efficient and therefore is a 'best practice' unit. This does not necessarily mean that this unit is efficient in an absolute sense; it signifies only that there are no other DMUs (or combinations of DMUs) operating more efficiently in the study. However, if $e_k^* < 1$, the unit is relatively inefficient, which implies that there are other DMUs (or combinations of DMUs) displaying greater efficiency and therefore there is room for improvement.

4) Model 2 identifies for each inefficient DMU k a reference set. This is the set of relatively efficient DMUs (i.e. those DMUs with $e_k^* = 1$) to which the inefficient unit has been most directly compared in determining its relative efficiency score. Thus, DEA avoids the need to explore all DMUs to understand the nature of the inefficiencies of the DMU in question (Thanassoulis *et al.*, 1987).

For computational convenience, it is usually the dual of model 2 that is solved in practice. Despite providing the same information as the primal model, the dual is of great use since it sheds light into the real nature of the DEA methodology by looking at the same problem from another perspective. Using Z_k, l_j, s_r^+, s_r^- as dual variables and assigning each one to the constraints of model 2, the dual formulation is thus:

[2] In this respect, Charnes *et al.* (1994, p. 6) noted, "…the weights (multipliers) are to be selected in a manner that calculates the Pareto efficiency measure of each DMU…". Simply defined, a DMU is Pareto efficient when no other DMU (or combinations of DMUs) can allocate inputs and outputs to achieve a better solution.

$$\min \qquad e_k = Z_k - e\left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right)$$

subject to:

$$\begin{aligned}
&\sum_{r=1}^{s} l_j x_{ij} = x_{ik} Z_k - s_i^- \quad ; \quad i = 1,2,\ldots,m \\
&\sum_{r=1}^{s} l_j y_{rj} = y_{rk} + s_r^+ \quad ; \quad r = 1,2,\ldots,s \qquad (3) \\
&l_j, s_r^+, s_i^- \geq 0, \quad Z_k \ \textit{unconstrained}
\end{aligned}$$

The interpretation of the dual is as follows:

1) For each DMU k being evaluated, the dual tries to form a composite unit operating more efficiently than DMU k. More precisely, a composite unit is an imaginary efficient unit constructed from a DMUs reference set, providing a set of targets for an inefficient unit. The first constraint forces the composite unit to produce outputs that are at least equal to the corresponding outputs of DMU k. The second constraint finds out how much less proportion Z_k of the inputs of DMU k the composite unit needs.

2) The intensity factor Z_k indicates the proportional reduction required in all inputs of DMU k in order to become efficient. On the other hand, the slacks s_r^+ and s_r^- represent the additional increase in outputs and/or decrease in inputs (after implementation of the Z_k reduction) to make DMU k efficient. As a result, a DMU k is efficient if and only if no composite unit outperforming k can be found, i.e. $Z_k^* = 1$ and $s_r^{+*} = s_r^{-*} = 0$.

3) Given the considerations above, the role of the dual weights l_j can be easily interpreted. l_j describes the proportion attributed to DMU k used to define the composite unit.

Banker *et al.* (1984) (BCC) modified the CCR model to estimate technical efficiency and allow for variable returns to scale[3]. This is obtained by introducing the following convexity constraint in model 3:

$$\sum_{j=1}^{n} l_j = 1 \quad ; \quad j = 1,2,\ldots,k,\ldots,n$$

[3] Returns to scale is a concept from economics that is related with increasing or decreasing efficiency based on size. Variable returns to scale consists of either increasing returns to scale (i.e., an increase in all inputs results in a proportionately greater increase in outputs) or decreasing returns to scale (i.e., an increase in all inputs results in a proportionately smaller increase in outputs).

The adjunction of this constraint simply means that each composite unit is a convex combination of the DMUs in the reference set. In other words, the convexity constraint ensures that the composite unit is of similar size as the DMU being evaluated.

It should be noted that there are other ways to formulate the previous models. The formulations considered above are known as input-oriented models, i.e., they focus on achieving efficiency via proportional reduction of inputs, while outputs remain constant. These models can be formulated as output-oriented if the emphasis is on achieving efficiency through the proportional increase of outputs, while holding the inputs unchanged. See Ali *et al.* (1995) for the output-oriented version of the basic DEA models.

4. Some Extensions, Software and Applications of DEA

Several authors developed various extensions and enhancements to the basic DEA models. Färe and Lovell (1978) proposed non-radial measures of technical efficiency. Charnes *et al.* (1982, 1983) described multiplicative models providing a piecewise log-linear or Cobb-Douglas envelopment. Other authors (Deprins *et al.*, 1984; Tulkens, 1993) developed free disposal hull models, which introduce non-convex piecewise linear production sets. Charnes *et al.* (1985a) presented the window analysis technique, allowing efficiency measurement of a set of DMUs across time. Charnes *et al.* (1985b) introduced the additive model, which constructs an empirical production function based on Pareto optimality. Färe *et al.* (1985, 1994) modified the basic DEA models to incorporate the concept of weak disposability of inputs and outputs. Tone (2001) suggested a slacks-based measure of efficiency in DEA.

The original DEA models have also been modified to handle different types of factor data. Banker and Morey (1986a) incorporated non-discretionary factors (i.e., factors beyond management control such as years of operations, advertising budget allocated etc.) into basic DEA models. Banker and Morey (1986b) extended this analysis to include categorical factors (dummy variables), which indicate the presence or absence of an attribute (e.g., a drive-in facility) or take a discrete set of values (e.g., quality of research output in a university). Other authors (e.g., Cook *et al.*, 1993; Cooper *et al.*, 1999a; Zhu, 2002) investigated the use of imprecise factor data (such as bounded, ordinal and ratio-bounded data). Scheel (2001) reviewed the inclusion of undesirable factors (such as waste and pollution) in DEA models.

Other important extensions are the inclusion of prior expert opinion or managerial preferences (e.g., Golany, 1988; Thanassoulis and Dyson, 1992; Zhu, 1996a; Thanassoulis and Allen, 1998; Halme *et al.*, 1999) and the use of weight restriction approaches (e.g., Dyson and Thanassoulis, 1988; Charnes *et al.*, 1990; Thompson *et al.*, 1990; Li and Reves, 1999; Despotis, 2002). Section 5.2 discusses some of these proposals in more detail. For useful reviews on these extensions see Allen *et al.* (1997) and Pedraja-Chaparro *et al.* (1997).

To support the user through the modelling process, numerous software tools have been developed. Table 1 displays a limited list of these tools and their modelling features; for a more comprehensive list see, for example, Emrouznejad (1995-2001). Some of them are commercial tools (e.g., Frontier Analyst), while others were developed for educational and research purposes (e.g., SEM). Their prices can range from the as expensive as £2400 (Frontier Analyst with capacity to analyse 2500 units) to the freely available software. Needless to say, the price tag tends to go in hand with the units capacity feature (except for free software). From the list below, DEA Excel Solver seems to be the most complete in terms of modelling features; Frontier Analyst has better graphical features but is the most expensive. Overall, it can be said that these tools support the principal DEA models and extensions, but they are usually slow to incorporate more recent developments, which can sometimes make a real difference to the modelling process.

[Insert table 1 about here]

Before starting to discuss some of the issues related with DEA, it is opportune to illustrate its uses through examples of applications (see Table 2). The bulk of the applications reported so far concentrate on the public sector. Indeed, in their original study, Charnes *et al.* (1978, p. 429) emphasised that their analysis was "...centered on decision making by not-for-profit entities...". DEA suits these situations specially because the presence of qualitative data makes it difficult to use traditional efficiency techniques. However, as Norman and Stoker (1991) demonstrated, there is no reason why DEA should not be equally applicable to private-sector organisations, in particular, where problems of measurement and comparability persist. Perhaps the realisation of this has lead to an increasing number of private sector applications being reported in recent times.

[Insert table 2 about here]

5. STATE OF THE ART ISSUES IN DEA

The previous sections showed that DEA is a powerful tool to measure performance and presents clear strengths over other approaches. However, one should be aware that the application of DEA also involves the recognition of peculiar difficulties.

5.1 Factor Selection

Which factors should be included in the model? The literature (e.g., Charnes *et al.*, 1994) recommends that all factors affecting the performance of the DMUs being evaluated should be included. The role of the analyst is to reduce this initial number of factors to the most relevant ones. In addition, there should be a relationship (whether theoretically, experientially or statistically) between inputs and outputs eliminating, if necessary, redundant factors. The availability of data and experience of managers also form part of the basis for variable selection.

At present, there are no formal criteria for selecting the appropriate input-output variables or for measuring their explanatory power. This is not surprising since DEA does not impose any functional form to the factor relationships, and, as a result, hypothesis testing is impossible without making further assumptions on the distribution of the residuals. Recognising this problem, Sexton *et al.* (1986) conducted a sensitivity analysis and their results showed that the efficiency scores may be greatly sensitive to factor selection, particularly, if they "affect the shape and position to the efficiency frontier in the neighbourhood of specific DMUs" (pp. 86-87). They went on to argue that any DMU could virtually maximise its efficiency through the manipulation of factors.

Several studies have attempted to address this issue. Thanassoulis *et al.* (1987) claimed that a useful procedure to pin down the appropriate factor set is to conduct a series of regression analysis. Oral and Yolalan (1990) argued that it may be meaningful to consider several runs with different combinations of factors. Norman and Stoker (1991) proposed a trial and error procedure called 'stepwise approach' based on correlation analysis to determine the final set of factors. Bates *et al.* (1996) recommended a procedure comparable with the stepwise regression approach. Other approaches include, for example, the use of principal component analysis (e.g., Adler and Golany, 2001) and multivariate statistics (e.g., Jenkins and Anderson, 2002) to remove highly correlated variables from the models. However, as several authors (e.g., Smith and Mayston, 1987; Dyson *et al.* (2001) demonstrated, the omission of such variables can have unpredictable impacts in the efficiency scores.

To sum up, it should be noted that in most real applications the choice of the factors is still done in an arbitrary or intuitive way. Should these be selected using regression analysis, trial and error procedures, managers' knowledge or other statistical methods? Researchers have been producing some interesting work, but a common agreed approach is still illusive.

5.2 Discriminating Power

There are often cases where DEA fails to discriminate between the DMUs and, as a result, all or most of them are rated with unit-efficiency scores. This problem may occur because the number of DMUs is too small when compared with the number of factors[4]. Another related reason is that some DMUs assign excessively high weights to their most favourable factors, while the remaining inputs and outputs are simply ignored. This is clearly unsatisfactory since it creates biased efficiency scores.

The literature reports various models and methods to tackle the discriminating power issue. One possible approach is to incorporate preferential information into DEA models. Golany (1988), Thanassoulis and Dyson (1992) and Zhu (1996a) proposed the specification of preferential information through the selection of desirable input and output targets. Other authors suggested the use of hypothetical DMUs (e.g., Thanassoulis and Allen, 1998) and value efficiency analysis (e.g., Halme *et al.*, 1999) to capture value judgments. The weakness of this approach is that it requires additional subjective information from decision makers.

Another approach is to impose restrictions on weight values. Dyson and Thanassoulis (1988) suggested a method to impose lower bounds on the weights for the case of a single input. Charnes *et al.* (1990) tackled the problem by requiring virtual multipliers to belong to given closed cones. Thompson *et al.* (1990) applied the assurance region method to place bounds on the weights. Golany and Roll (1994) showed how the concept of 'standard' DMUs could be used to set controls on the weights. Li and Reves (1999) proposed a multiple criteria DEA to impose bounds on the weights. Despotis (2002) discussed the globally efficiency approach to restrict weight flexibility. One drawback of limiting weight flexibility is that the model may become infeasible if the bounds are too tight. Several studies (e.g., Podinovski and Athanassopoulos, 1998; Podinovski, 2001) provide evidence that weight restrictions can also underestimate efficiency scores.

An alternative remedy is to construct a matrix of cross-efficiencies (e.g., Sexton *et al.*, 1986, Doyle and Green, 1994), i.e., a table where the relative efficiency of each DMU is calculated based on the optimal weights chosen by the other DMUs. Thus, if the majority of the DMUs rate a particular DMU with very high efficiency scores, then this DMU is likely to be efficient. Conversely, if it is rated with low efficiency scores, then it is likely to be an

[4] Charnes and Cooper (1991) recommended that the minimum number of DMUs should be at least equal to three times the sum of the factors, while Dyson *et al.* (2001) suggested that the number of DMUs should be at least twice the product of the number of inputs and number of outputs.

inefficient DMU. The cross-efficiency approach is also used to rank both efficient and inefficient DMUs, a topic on its own open to debate (e.g., see Adler *et al.*, 2002).

Finally, the lack of discriminating power can be addressed by cluster analysis (e.g., Golany and Roll, 1989; Ganley and Cubbin, 1992; Athanassopoulos and Balantine, 1995). Here the objective is to detect different subgroups of DMUs (clusters) which are operating under similar circumstances and, therefore, to gain insight about the differentiating features among them. As an illustration of a cluster analysis application, consider a NATO burden-sharing investigation described in Kim and Hendry (1998). After a trail run with 18 factors the authors realised that 14 of the 16 member nations of NATO had a net-burden index of 100%. The problem was tackled by considering two different approaches: the stepwise approach and the weight constraint approach. These approaches were, then, balanced with the incorporation of cluster analysis in order to constitute separate clusters of NATO members with similar net-burden indexes. A factor that possibly reduces the usefulness of this technique is that there may be difficulties in the definition of the cluster.

These approaches undoubtedly show valuable progress; however, one should not jump to immediate conclusions since most of the cases, which they describe, incorporate a high degree of subjective factors and ambiguity. Although the inclusion of bounds on weights is widely recognised in DEA studies there remains some dispute as to how incorporate them into the models without significant "side effects". Hence, further research is needed to eliminate such factors and issues like where to set bounds and how to determine their range are yet to be resolved at an adequate level.

5.3 Stochastic Aspects

How robust are efficiency scores to errors in data? How do outliers affect the efficiency frontier? The sensitivity of DEA to errors in data poses a different problem. Retzlaff-Roberts and Morey (1993, p. 379) stated that a primary limitation of classic DEA models is that "they are deterministic and have no means of allowing for uncertainty". In fact, since DEA relies on a search for extreme points to construct the efficiency frontier, it could be contaminated by outliers (atypical data) and data errors (e.g., measurement errors, misreported or miscoded data, etc.). The result is that the efficiency scores for some, or possibly all, DMUs may be seriously distorted.

For instance, Sexton *et al.* (1986) argued that errors in data could affect DEA results in one of the two following ways: if the error happens in an observation which is on the efficiency frontier, the resulting error is likely to produce large changes on the efficiency scores, while if the error occurs in an observation related to an inefficient DMU, the resulting error will be only restricted to that DMU. In this respect, Wilson (1995) suggested that data entry errors or measurement errors should be corrected (if possible) or deleted from the analysis (if data-checking is expensive and resources are limited).

Several researchers have made proposals to integrate stochastic influences into DEA. Some of this research falls into the scope of linear programming (LP)-sensitivity analysis and it examines the robustness of the efficiency scores in response to variations in factor data. This is the case of Charnes *et al.* (1985a) and Charnes and Neralic (1990), who described the theoretical foundations of DEA-sensitivity analysis. Thompson *et al.* (1994) presented a LP-sensitivity analysis model from the standpoint of duality that permits simultaneous variations in all data. These approaches, though, are limited in their applicability to the simplest cases. Other more recent methods compute stability regions, by means of modified CCR models, under which a given DMU remains efficient to factor variations (e.g., Zhu, 1996b; Seiford e Zhu, 1998; Zhu, 2001). Cooper *et al.* (2001) provide a comprehensive evaluation of recent developments in DEA-sensitivity analysis.

A different stream of research discusses stochastic formulations of DEA models so as to deal with random variations in factor data. Banker (1993) provides the statistical foundations for the DEA models with a single output and multiple inputs. Olesen and Petersen (1995) developed a chance-constrained model that attempts to estimate the sensitivity of efficiency scores with respect to an unknown amount of noise in data. Recent enhancements on the chance-constrained approach are described in Cooper *et al.* (1998) and Huang and Li (2001). Post (2001) proposed a mean-variance framework derived from the theory of stochastic dominance to incorporate factor uncertainty. However, a common criticism of these models is that they require additional assumptions; in particular, factor data must be drawn from probability distributions and the model may need to be solved as a non-LP problem.

Other authors have investigated methods to detect outliers in DEA models and assess their impact on efficiency measurement of the remaining observations. Gstach (1998) put forward a stochastic approach called DEA+ that enables to filter outliers. Pastor *et al.* (1999) described a simple statistical test to determine whether a given observation causes statistically significant changes in the remaining efficiency scores. Ondrich and Ruggiero (2002) discussed the use of the jack-knifing technique to detect influential observations. Some of these techniques, though, may require additional assumptions regarding the distribution of noise and may not be applicable to all cases.

Regardless of the method applied, the state of the art of stochastic issues, as Olesen (1995, p. 30) noted, “is not developed to a satisfactory level of generality and applicability”. Despite some challenging and promising proposals, the investigation is still at an incipient stage of development.

5.4 Other Issues

This section discusses other, but by no means less important, issues that have been drawing the interest of DEA researchers.

Returns to scale

One of the most debated DEA topics in recent times has been the identification of the nature of returns to scale, i.e., whether the underlying technology exhibits increasing, constant or decreasing returns to scale. After the initial breakthrough made by Banker (1984) and Banker *et al.* (1984), many other papers followed with alternative proposals or enhancements to existing models (e.g., Färe *et al.*, 1985; Banker, 1996; Golany and Yu, 1997; Seiford and Zhu, 1999; Simar and Wilson, 2002). This is not surprising since the use of a constant returns to scale model in a production technology with underlying variable returns to scale (and vice versa) can lead to biased efficiency scores. However, work is far from over; issues such as alternate optima, quantification of the magnitude of returns to scale, determination of local returns to scale for both efficient and non-efficient units, scale independent models, etc. continue to enthusiastically engage many researchers.

Environmental factors

DEA models assume that environmental conditions are homogenous across DMUs. In practice, though, this assumption is rarely satisfied. To avoid biased efficiency scores, researchers try to capture environmental variations across DMUs by incorporating environmental factors (also called non-discretionary or exogenous) in DEA models. Examples of such factors are social and economic considerations, market dimension, branch age, location quality, competitor strength, etc. However, as Smith and Mayston (1987) pointed out, these variables require careful handling because of their subjective nature. How to define and measure them? Should be they considered as inputs or outputs? How do they affect efficiency scores? Apart from a few papers (Banker and Morey, 1986a; Ruggiero, 1998; Fried *et al.*, 2002; Haas and Murphy, 2002), there is no study attempting to systematically address these questions.

Undesirable factors

The insertion of undesirable factors in DEA models is another topic worthy of further research. Examples of undesirable factors are waste and pollution (on the output side) or waste that re-enters the production process (on the input side). Traditional DEA models often assume that desirable outputs should be maximised while desirable inputs should be minimised. When undesirable factors are present, though, undesirable outputs and inputs should be minimised and maximised respectively. Different schemes to handle undesirable factors have been proposed; a helpful review can be found in Scheel (2001). The problem of incorporating such factors in DEA models is that, as Dyson *et al.* (2001) and Scheel (2001) demonstrated, existing approaches produce different impacts on performance measures. Future work should clarify the pros and cons of the different handling schemes so that users can make an informed selection on a given application.

Computational efficiency

The increasing use of DEA in large-scale and complex applications raises an additional issue related with computational efficiency and robustness. Recently, a number of research articles (e.g., Ali, 1994; Olesen and Petersen, 1996; Barr and Durchholz, 1997; Dulá and Thrall, 2001) brought this problem to attention and emphasised some devices that permit efficient DEA computation. Latest developments in parallel computing and the advent of the Internet created new opportunities for accelerating DEA computations. Such opportunities, though, remain relatively unexplored to date.

Dynamic DEA

There has been a rising interest in using DEA to measure efficiency variations over time. Examples of dynamic DEA approaches are the window analysis technique (Charnes *et al.*, 1985b), the network technology method (Färe and Grosskopf, 1996), the intertemporal cost minimisation framework (Sengupta, 1999) and the slack-adjusted DEA model (Sueyoshi and Goto, 2001). However, an important snag of current methods concerns their limited practical usability. As Ganley and Cubbin (1992, p. 158) stated, "[dynamic DEA] produces little more than a continuum of 'static' results". Hence, future research efforts need to concentrate on developing practical methods for measuring efficiency over time. Several issues also need to be carefully considered including, for example, the detection of trends and seasonal effects over time, incorporation of inflation considerations and treatment of capital inputs.

Visualisation of results

Another field where little research has been done relates to the development of effective ways for visualising the results obtained from DEA models. How to best graphically display the efficiency frontier when there is a considerable amount of factors in the analysis? This is a crucial question because the success of DEA largely depends on an adequate understanding of the results by managers. One approach due to Desai and Walters (1991) is to reproduce the efficiency frontier using a parallel axis representation. El-Mahagary and Lahdelma (1995) presented a technique based on various two-dimensional bar charts that convey the fundamental information of the efficiency frontier. Although these are interesting approaches, there is a lack of comprehensive studies which identify simple methods for displaying DEA results with several factors.

Profit efficiency

The theme of profit efficiency also deserves more attention from the DEA community. In private sector applications it may be more appropriate to measure the potential for maximising profit than to measure efficiency. However, this is far from straightforward because organisations face diverse environments. Indeed, a DMU realising lower profit may not be necessarily less efficient than a DMU with higher profit due to an unfavourable environment. Furthermore, profitability can be measured using different alternative indicators including, for example, return on investment and return on sales. Only recently the measurement of profit efficiency has received attention from researchers (e.g., see Athanassopoulos and Thanassoulis, 1995; Zhu, 2000; Tone, 2001), yet a common approach still needs to be established. A more intense application of DEA to the private sector may depend on adequately addressing this issue.

Model choice and quality

From this review it can be inferred that there is a large number of DEA models and extensions. This, in turn, can be a heavy burden both for the novice modeller and for the experienced user. The development of a taxonomy of existing DEA models and extensions would seem to ease the work of modellers. The concept of a generalised DEA model (e.g., see Gang *et al.*, 1996) also appears to be a sensible idea, but research is still ongoing. Apart from the overwhelming quantity of models, issues such as how to select a model and the extent to which a model is adequate for a given application are crucial. Unfortunately, because DEA has no statistical tests, these decisions are currently made on an *ad hoc* basis. The few existing papers (e.g., Smith, 1997; Pedraja-Chaparro *et al.*, 1999) have approached model quality through sensitivity analysis and simulations. Given the importance of the topic, there is a surprising lack of research.

6. CONCLUDING REMARKS

The main contribution of this paper is that it gives a useful review of an efficiency measurement approach called Data Envelopment Analysis and highlights some key issues that are the focus of ongoing research. Given the fast paced and fragmented nature of the field, it is sensible now and then to make an assessment of the developments accomplished and point the way forward – and this is what was attempted here. Furthermore, it offers a useful starting point for would-be DEA researchers and modellers. Indeed, the number of candidate models and the number of issues to consider when applying DEA can be overwhelming for any modeller. This paper discussed some of those models and issues and pointed to additional resources as appropriate.

In light of this review, it seems reasonable to state that the measurement of efficiency has been enriched by the DEA literature, which provides renewed insights into the field. In particular, DEA is a meaningful approach based on linear programming for measuring the relative efficiency of organisational units in situations where the presence of several inputs, outputs and qualitative data prevent the use of traditional efficiency/productivity techniques.

However, this paper has shown that DEA suffers from several unresolved issues, which can have devastating consequences on the computation of the efficiency scores and may vitiate the interpretation of results. What was also apparent from this review is that it requires considerable skills and thus its use may still be the province of specialists. Indeed, it is not uncommon to find published papers with methodological flaws! Researchers should perhaps consider ways to make DEA user-friendlier. The development of powerful software may be a sensible thing to pursue, but still it is not without its dangers.

Hence, it seems also sensible to state that unless these issues are addressed to an adequate level of applicability it may be difficult to accept DEA as an established and reliable approach to efficiency measurement. As demonstrated in this paper, the literature is beginning to provide important steps towards resolving some of these problems. Meanwhile, though, DEA should be used prudently.

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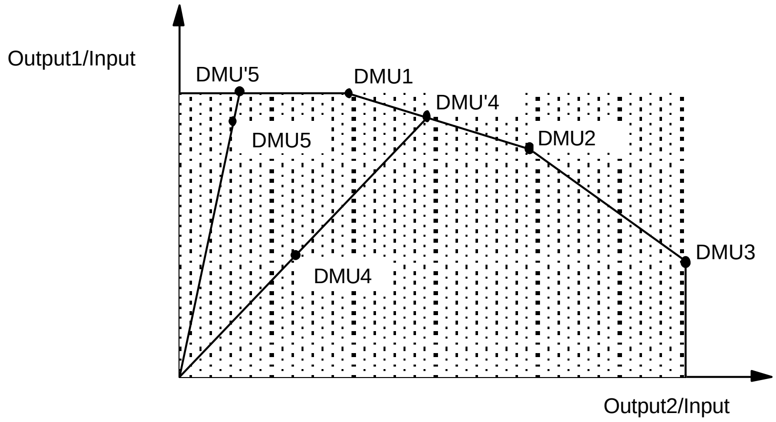

Figure 1. The DEA methodology for the single input two-output case.

Table 1. An Illustration of DEA software tools.

SOFTWARE	WEBSITE	MODELLING FEATURES SUPPORTED
DEA Excel Solver	http://www.deafrontier.com/ Trial version available	• CCR, BCC, multiplier, non-radial, slack-based, measure-specific • Input/output orientation and weak disposability • Constant and variable returns to scale • Super efficiency, weight restrictions and preference structure models • Sensitivity analysis • Cost, revenue and profit efficiency • Environmental and undesirable factors • Free Disposal Hull • Malmquist indexes
Frontier Analyst	http://www.banxia.com/famain.html Trial version available	• CCR and BCC models • Input/output orientation • Constant and variable returns to scale • Weight restrictions • Cross-efficiencies matrix
SEM – Efficiency Measurement System	http://www.wiso.uni-dortmund.de/lsfg/or/scheel/ems/ Free for academic purposes	• CCR, BCC, additive, radial, slack-based, measure-specific • Input/output orientation • Constant and variable returns to scale • Super efficiency and weight restrictions • Non-discretionary factors • Window analysis • Free Disposal Hull • Malmquist
Warwick DEA Software	http://www.deazone.com/software/ Trial version available	• CCR, BCC, additive, non-radial and mixed target models • Constant and variable returns to scale • Super efficiency and weight restrictions • Non-discretionary variables

Table 2. An Illustration of Public and Private Sector Applications.

INDUSTRY SECTOR	SAMPLE REFERENCES	USAGES
Education	Ray (1991)	Resource use efficiency
	Sarrico and Dyson (2000)	Performance measurement in UK universities
	Portela and Thanassoulis (2001)	Assessment of pupil efficiency
Health services	Desai *et al*. (1994)	Site selection and spatial efficiency of mental health clinics
	Chilingerian (1995)	Evaluation of physician efficiency
	Athanassopoulos and Gounaris (2001)	Assessment of technical and allocative efficiency of hospital operations
Financial services	Cummings *et al*. (1999)	Measurement of cost and revenue efficiency in the US life insurance industry.
	Athanassopoulos and Giokas (2000)	Assessment of bank branch efficiency
Manufacturing	Thore *et al*. (1996)	Management of the product life cycle in the US computer industry
	Chandra *et al*. (1998)	Efficiency evaluation of Canadian textile companies
	Talluri *et al*. (2000)	Selection and evaluation of flexible manufacturing systems
Agriculture	Haag *et al*. (1992)	Assessment of the relative technical efficiency of agricultural production units
	Kao and Yang (1992)	Efficiency measurement of forest districts for reorganisation
Retail	Banker and Morey (1993)	Design and operational decisions of fast food outlets
	Thomas *et al*. (1998)	Assessment of retail store efficiency
Energy and heavy industries	Ray and Kim (1995)	Evaluation of cost efficiency in the US steel industry
	Thompson *et al*. (1996)	Efficiency and profitability measurement of oil companies
Transportation	Sarkis (2000)	Evaluation of operational efficiency of US airports
Other	Lovell *et al*. (1995)	Measurement of macroeconomic performance of OECD countries
	Kim and Hendry (1998)	NATO burden-sharing investigation
	Zhu (2000)	Profitability measurement of Fortune 500 companies