Robust Optimisation with Xpress-Mosel

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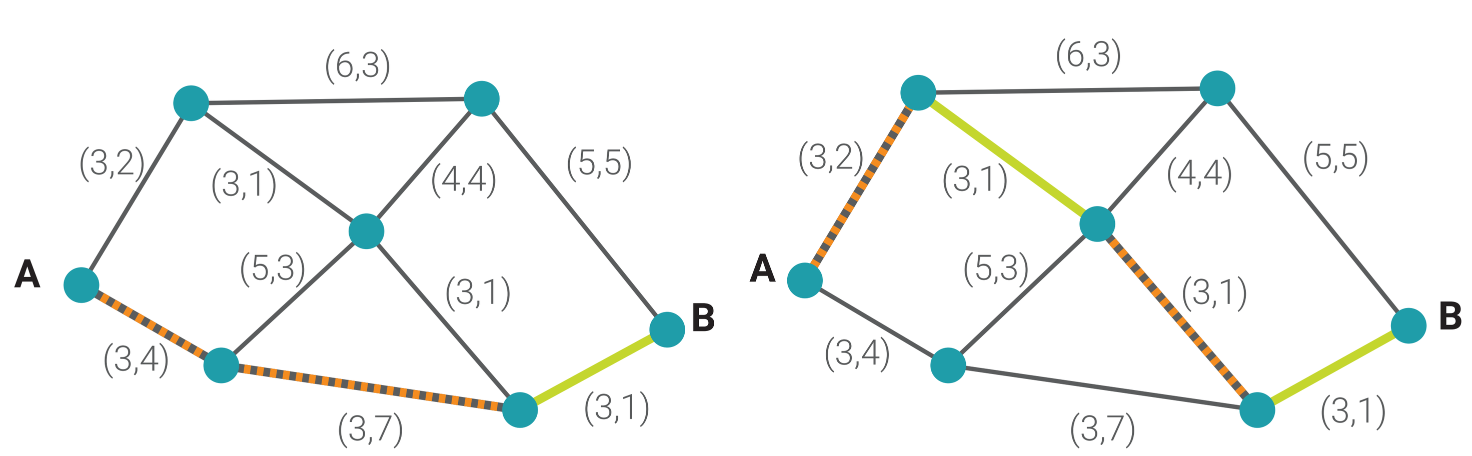
What Is Robust Optimisation?

Robust optimisation is a modelling paradigm for problems that are described with a degree of **uncertainty**. Robust optimisation solves an uncertain optimisation problem by hedging against the **worst-case** scenario realisation of uncertainty.

Robust optimisation relies on the fact that often uncertainty can be capped (for instance using confidence estimation). Robust optimisation finds a solution that is guaranteed to be feasible for **any** value of the parameters within the uncertainty set.

Example: the robust shortest path. Find the shortest route from A to B on a road network where each road link has a traversal time that varies within a given range. Also, not more than **two** links can have an increased traverse time.



With no uncertainty and all travel times at their nominal value, the shortest path is the one on the left, with a total length of 9. However, it has a cost of 20, much higher than the robust one on the right (15), if the two longer links are maliciously placed.

Robust and stochastic optimisation. While stochastic optimisation uses a probabilistic distribution, robust optimisation does **not** take into account any probabilistic characterisation of uncertainty. It simply assumes that any scenario in the uncertainty set is possible.

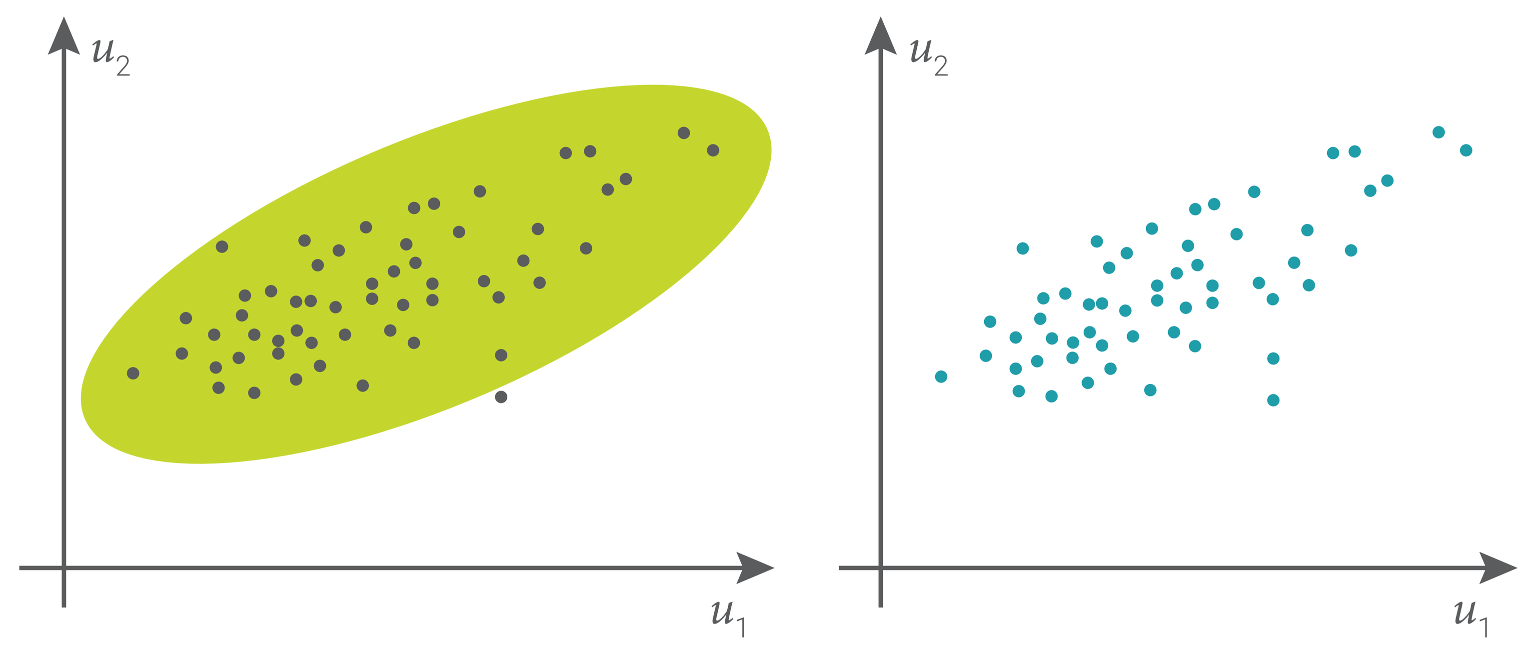
Modelling robust problems. Robust optimisation requires extra modelling skills. For instance, the uncertainty set must be **accurate**: if too conservative, the robust solution is poor; if uncertainty is too small, the solution is good but might not be robust. But we must ask other questions:

- How can uncertainty be formulated?
- What are the values of the uncertain parameters in a robust solution?
- What is their meaning?

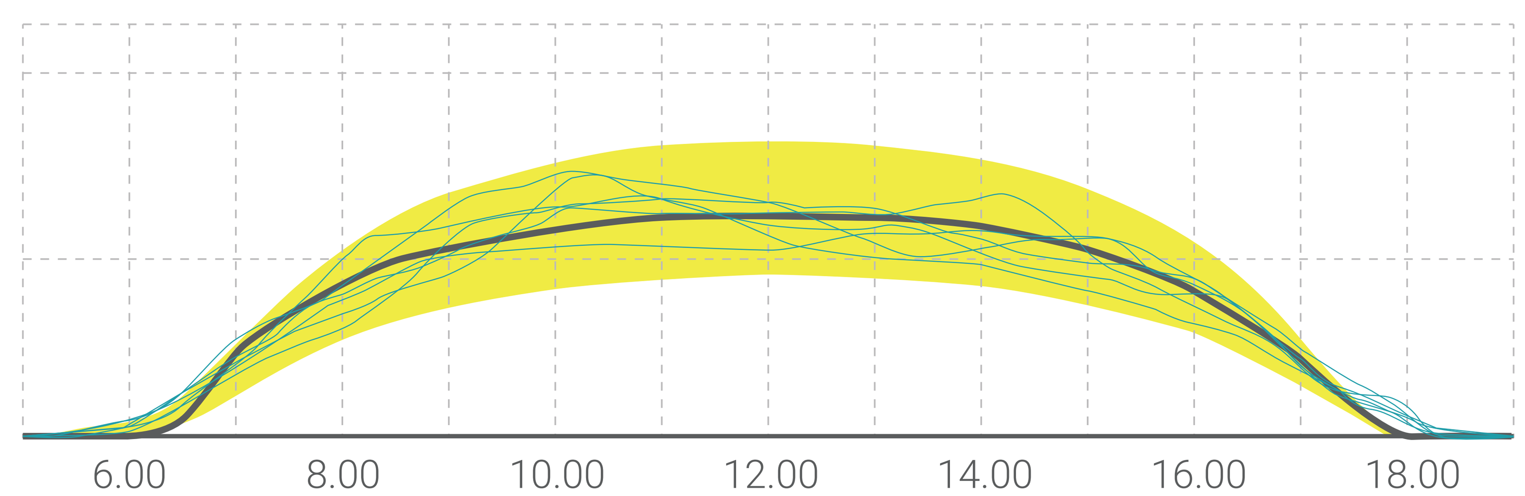
Two Real-World Applications

Marketing Campaign Optimisation. Propensity scores indicate the willingness of a potential customer to purchase. Using regression models, percentile, variance, and model quality can be extracted.

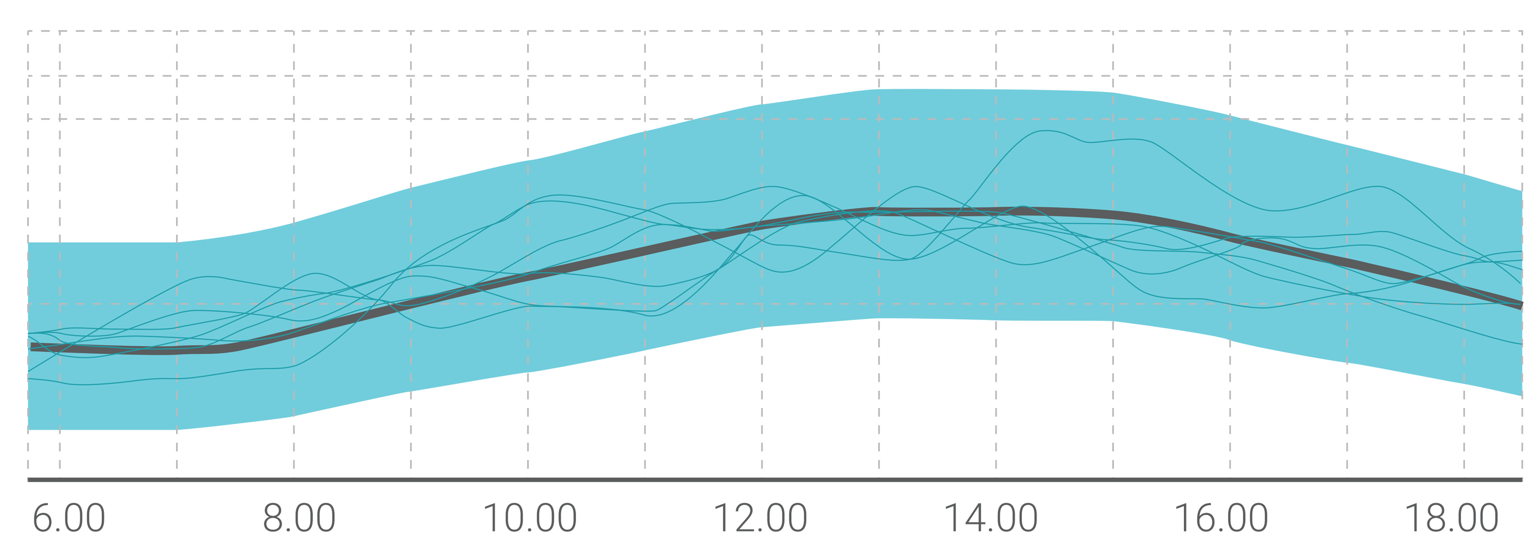
Using this statistical data, we create an uncertainty set for all propensity score vectors: it is a confidence ellipsoid that uses mean/covariance data from this model.



Unit Commitment with Renewables. Optimal use of renewable energy management is a problem of enormous impact. It is also inherently uncertain: daily amount of renewable energy (both solar and wind) can vary significantly owing to climatic conditions.



Uncertainty in solar (above) and wind (below) energy.



The list of real-world problems goes on: production planning, portfolio optimisation, telecommunication network design, to name only a few.

Mosel for Robust Optimisation

Xpress-Mosel has a module for robust optimisation that allows for quick implementation of a robust optimisation model. The module handles three types of uncertainty:

Polyhedral: The uncertainty is a set of linear constraints, e.g. requiring the sum of uncertainties to be less than a given threshold.

Scenarios: The solution is robust against a set of vectors of **historical** values of the uncertain parameters and their combinations.

Ellipsoidal: The uncertainty set is the confidence ellipsoid centred at a given vector and with a given correlation matrix. This is especially useful with the scenarios uncertainty when using **clustering** techniques.

```
declarations
  ITEMS = 1..5                     ! Index set
  CAPACITY: integer                ! Scalar constant
  PROFIT: array(ITEMS) of real     ! Profit data
  RES: array(ITEMS) of real        ! Resource use (nominal)
  Epsilon: real                    ! Confidence level
  u: array(ITEMS) of uncertain     ! Uncertainty
  x: array(ITEMS) of mpvar         ! Decision variables
end-declarations

forall(i in ITEMS) x(i) is_binary
sum   (i in ITEMS, j in ITEMS) u(i)^2 <= Epsilon
sum   (i in ITEMS) (RES(i) + u(i)) * x(i) <= CAPACITY

maximize(sum(i in ITEMS) PROFIT(i) * x(i))

forall(i in ITEMS)
  writeln(i, ": ", getsol(x(i)), ", ", getsol(u(i)))
```

Tell us about your uncertain problems. Have you got an optimisation problem that is subject to uncertainty? Tell us about it at **robustoptimisation@fico.com**

References

1. Belotti, P., Csizmadia, Z., Heipcke, S., Lannez, S. Robust Optimisation with FICO Xpress: usage guidelines and examples.
2. Ben-Tal, A., El Ghaoui, L., Nemirovski, A. (2009). Robust optimisation. Princeton University Press.
3. Bertsimas, D., Sim, M. (2004). The price of robustness. Operations Research, 52(1), 35–53.
4. Lannez, S., Marketing Campaign Optimisation, presented at ROADEF 2015.

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