NEW YEAR'S RES-O.R.-LUTIONS

LOUISE MAYNARD-ATEM

Firstly I would like to say thank-you to Louise Orpin at the O.R. society for inviting me to speak at the Careers Open Day in Birmingham on 20 November.

I really enjoyed sharing my experiences with all of those who attended, and it was great to see so many people so passionate about a career in O.R. I also found it really fascinating to talk to the other speakers and exhibitors. Duncan Stewart from DSTL talked about how O.R. is being used in Defence and Michael Nicholson from IBM discussed the role of predictive analytics in online sports reporting and even though I have no interest in sport, I was actually tempted to watch a game of rugby afterwards.

An overarching take-home message for me is something I have said many times before, but really got to see it in action at the careers fair; the influence of O.R. really is everywhere – from your local Tesco to booking your next holiday with British Airways. The last slide from my talk in Birmingham posed the question of whether or not I felt I had made the right choice in moving from chemistry to O.R. and I came away from the day thinking an even more resounding yes!

New Year's Res-O.R.-lutions

January is always a time for people to set their personal goals for the year but sometimes professional goals can get neglected (unless aligned with a New-Year review process). Alongside my renewed commitment to actually use the gym membership I have been paying for, I would like to improve on my technical O.R. skills this year. Coming from a non-O.R. background, I am sometimes conscious of the fact that my in depth understanding of some O.R. techniques may not be at the same level as others in my position, and this is something I would like to change as it feel it would have both personal and professional benefits.

If you are anything like me, reading notes and textbooks is only effective up to a point. I find that I learn best by actually working through examples and attempting questions. This month, I would like to make good on my previous promise of putting specific techniques in the spotlight, giving worked examples and posing problems to be solved.

Problem Page

Technique - Linear Programming:

Linear programming (or linear optimisation) is a mathematical method for determining a way to achieve the most favourable outcome; companies want to maximise profits and minimise cost using limited resources therefore the technique is potentially very useful.

This type of programming consists of the following basic components:

- Decision variables – these represent the quantities we wish to determine.
- Objective function – this represents how the decision variables affect the cost or value to be optimised.
- Constraints – these represent how decision variables use the limited resources that are available.
- Data – quantifies the relationships represented in the objective function and the constraints.

In a linear program, the objective function and constraints are linear relationships, meaning that the effect of changing a decision variable is proportional to its magnitude. This provides a powerful analytical tool for supporting evidence-based decision-making.

Examples:

The following worked example relates to a more commercial scenario, where the use of linear programming/optimisation is prevalent:

1. Production Planning

   a. A company makes three products, in quantities x1, x2 and x3 per month.

   b. Profits per unit = 1.0, 1.4 and 1.6 respectively

   c. Each product uses different amounts of resources (labour and materials) as shown in the following table:

	Product 1	Product 2	Product 3
Labour	1/1000	1/800	1/500
Materials	1/1200	1/700	1/600

In order to maximise profits, what is the optimum production plan?

This problem can be solved as follows:

2. Maximise: $x_1 + 1.4x_2 + 1.6x_3$

Subject to: $(1/1000)x_1 + (1/800)x_2 + (1/500)x_3 \leq 1$

$(1/1200)x_1 + (1/700)x_2 + (1/600)x_3 \leq 1$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

3. Solution*: $x_1 = 462$

$x_2 = 432$

$x_3 = 0$

*This solution was found using Microsoft Excel.

The **Simplex Method** is a popular algorithm used for solving linear programming problems and relies on the idea that the maximum value of the object function will occur at a 'corner' of a bounded feasible region. Equations rather than inequalities must be used in order to find the border of the feasible region and therefore slack variables are introduced. These are variables that are added in order to transform an inequality constraint into an equation.

Commercial uses of linear programming/optimisation range from menu planning to optimise meal production, creating portfolios in investment companies and crew and flight scheduling for airline companies; in light of this I think it is important that all of us at the start of our career in O.R. have at least a basic grasp of the technique.

Limitations:

It is important to consider the associated limitations and caveats when using any technique, and in terms of linear programming there are several to be mindful of:

- Since the technique optimises over a discrete period of time, it often only solves static problems, rather than dynamic ones. In the example above, the solution tells you what you to make during the month in question but does not take into account what you made in the previous month, or what you will need to make in the next month.
- Having solved this example in Excel, I noticed the sensitivity of the technique in that small changes to the inputs could produce very large changes in outputs, thus the solutions given are not particularly robust.
- In the example above you will notice that the solution does not involve making any of Product 3, although it gives the largest profit (though the solution shows this to be outranked by the considerably larger resource requirement); this poses several new questions including is it worth keeping Product 3 in production at all, by how much would you need to increase the profit margin in order to make this product viable and by how much do you need to cut resources again to make it viable? These questions are all particularly important from a commercial perspective, if you are looking to not only maximise profit but also to meet consumer demand.
- The primary limitation of this technique is that relationships involved have to be linear, and although it is possible to 'linearise' some functions, this does not detract from the fact that some real-world phenomena are poorly modelled by straight lines and non-linear relationships would be more appropriate.

Problem:

Now that I have discussed the basics of the technique, given an example of what it can be used for and highlighted the limitations, I thought I would end on a problem you can tackle yourselves, just to get those brains working after the festive break:

J&M Winery make two jug wines, House Red and Premium Red, and two higher-quality wines, Cabernet Sauvignon and Zinfandel, which it sells to restaurants, supermarkets and off-licenses. House Red is a blend of 20% pinot noir grapes, 30% zinfandel grapes and 50% gamay grapes. Premium Red is 60% cabernet sauvignon grapes and 20% pinot noir and gamay grapes. J&M's Cabernet Sauvignon is 100% cabernet sauvignon grapes and its Zinfandel is 85% zinfandel grapes and 15% gamay grapes. Profit of the House red is £0.90 per litre, profit on Premium red is £1.60 per litre, profit on Zinfandel is £2.25 per litre and profit on Cabernet Sauvignon is £3.00 per litre.

This season, J&M can obtain 30,000 pounds each of pinot noir, zinfandel and gamay grapes and 22,000 pounds of cabernet sauvignon grapes. It takes 2 pounds of grapes to make 1 litre of wine. If the company can sell all that it makes, how many litres of the various products should J&M prepare in order to maximise their profit?

I'm currently working on answering this question so I'd be interested to see how your answers compare with mine. I'm also interested in whether or not there are other techniques that may have been appropriate for solving problems of this nature, and your thoughts on the usefulness of the technique in general.

Answers on a postcard…or rather in an email to LMaynardAtem@live.co.uk and don't forget to put **Inside O.R. January Problem Solving** in the subject line.

<OR>

> '… the influence of O.R. really is everywhere – from your local Tesco to booking your next holiday with British Airways.'

Y2OR: THE SOLUTION

LOUISE MAYNARD-ATEM

This month I shall be handing the reigns over to Richard Wood, a young O.R. Society member who will be discussing his O.R. experiences both within and outside of academia.

Before I do though, I would like to thank all of those who sent me solutions to last month's linear programming problem. I thoroughly enjoyed reading your submissions and matching them to my own. I had some positive feedback indicating that you found the article both interesting and useful so I'll be sure to make technique based articles a regular feature. I thought I would publish one solution in particular, submitted by Alexander Finlayson, a senior lecturer at Teeside University:

Thank you also to Mark Montanana, who sent in the same solution. Mark also pointed out that the model gives a zero answer for the premium red, as it is made largely from the most scarce grape (cabernet sauvignon), despite being the most expensive. This brings me back to the limitations of the technique that I mentioned in last months article; for example the technique does not take into account such factors as consumer demand.

Did your solution match Alex's and Mark's? Get in touch with your thoughts on the article and what techniques you would like to see demonstrated on future problem pages. As always, my email address is Louise.Maynard-Atem@dh.gsi.gov.uk and I'd love to hear from you, whether it's techniques you'd like to know more about, topics you think these articles should cover or if you want to take a leaf out of Richard's book and write a contribution yourself.

<OR>

		House Red	Premium Red	Cabernet Sauvignon	Zinfandel	Total Profit			
		x_1	x_2	x_3	x_4				
	Litres to Produce	27631.58	0.00	11000.00	7894.74	£ 75,631.58			
	Profit per Litre	£ 0.90	£ 1.60	£ 3.00	£ 2.25	LHS = litres used		RHS = max available litres	Raw Material Supply
Supply Constraints	Pinot Noir	0.20	0.20			5526.32	<=	15000	30000
	Zinfandel	0.30			0.85	15000.00	<=	15000	30000
	Gamay	0.50	0.20		0.15	15000.00	<=	15000	30000
	Cabernet Sauvignon		0.60	1.00		11000.00	<=	11000	22000
		Blending Parameters							