Optimisation stories in Logistics and Transportation featuring Python

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Professor of Applied Optimization in Operations Research @ Vrije Universiteit, Amsterdam

Wednesday, September 16, 2020

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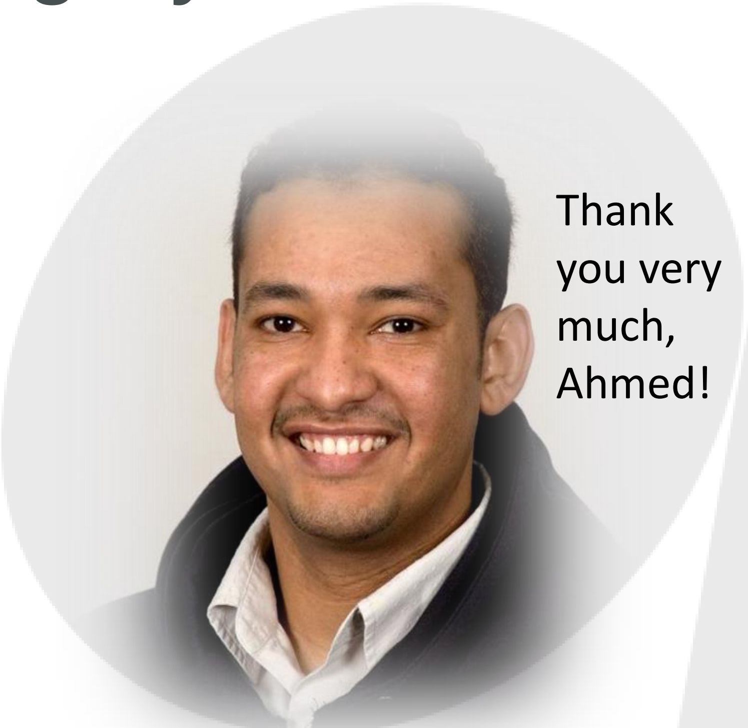

About me

Our "new normal" should not prevent acquainting…

joaquim.gromicho@ortec.com

Joaquim Gromicho, Portugal 1965-…

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Dutch Works Council

Education Factory

The Analytics Academy

Professor of Business Analytics at the Amsterdam Business School, UvA.

Editor in Chief of STAtOR, the 'glossy' of the Netherlands Society of Statistics and Operations Research.

Member of the **steering committee** of the EURO working group on the Practice of Operations Research.

EWG POR
PRACTICE OF OR

About this tutorial

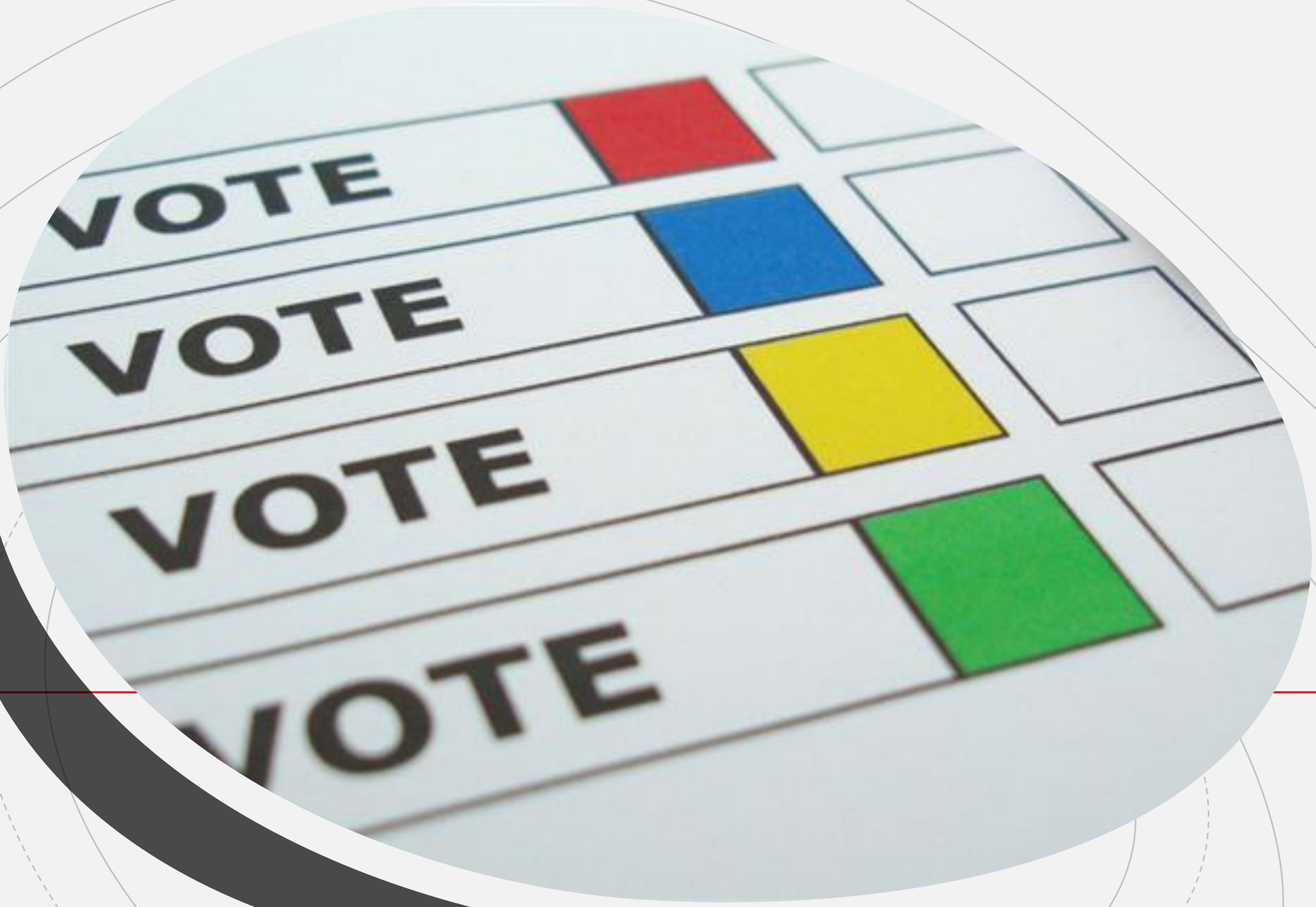
In one minute you will know if you made the right choice…

Main objectives

- Appealing to both the **curious** and the **trained** professionals.
- Highlighting the **value** of optimization and the **magic** op proven optimality!
- Learning how to model a business situation in a way that **enables** optimization.
- Using python as a **modeling** and resolution **tool**.
- **Understanding** and **interpreting** solutions.
- Illustrating a **methodology** to deal with uncertainties.
- Intentionally not too much focus on **routing.**
- **Relax**! The tempo will be high, but you may ask me the material!

Resources

- There is a **resources** section at [the schedule page] of this tutorial.
- The following URL should download the notebooks: http://bit.ly/JG-notebooks



A first poll

Are you now thinking about which session to move to?

Meet Alice

- Alice receives a beautiful rose.
- She cherishes it very much!
- She immediately searches an appropriate vase…

A glass is found!

- Alice has nothing but a lemonade glass at hand.
- It does seem suitable for a single, long stemmed, beautiful rose!

Unstable at first

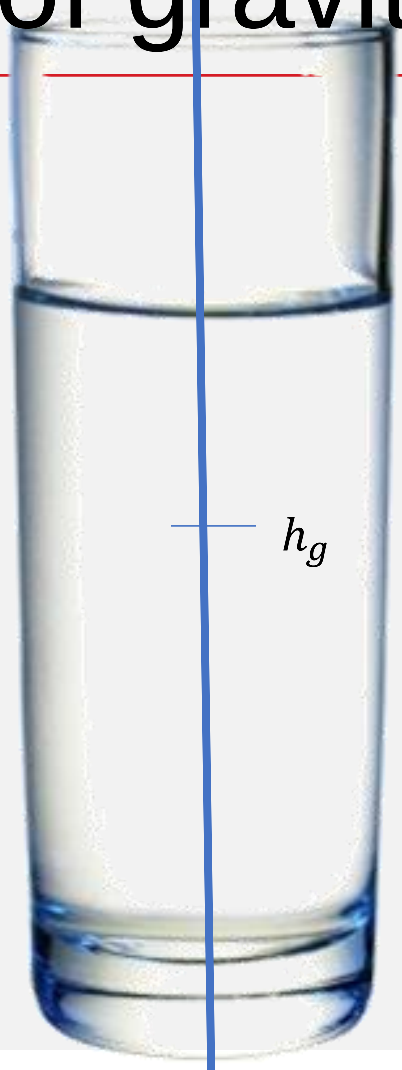
- When Alice places the rose in the glass, it tumbles, and she manages to hold it *just in time before it falls*!
- Alice is a clever girl, and she realizes that water will help, not only the rose self, but also the stability of the ensemble.
- And yes, it does! A bit of water keeps the glass stable.
- Then Alice thinks: let's be **very safe** and **fill up** the glass!
- Unfortunately, it become just as unstable as in the beginning… but much more wet!

Alice has a challenge to solve…

- An empty glass was unstable.
- A glass with some water was stable!
- A glass full of water was unstable again…
- What is the **optimal** level of water in terms of stability?

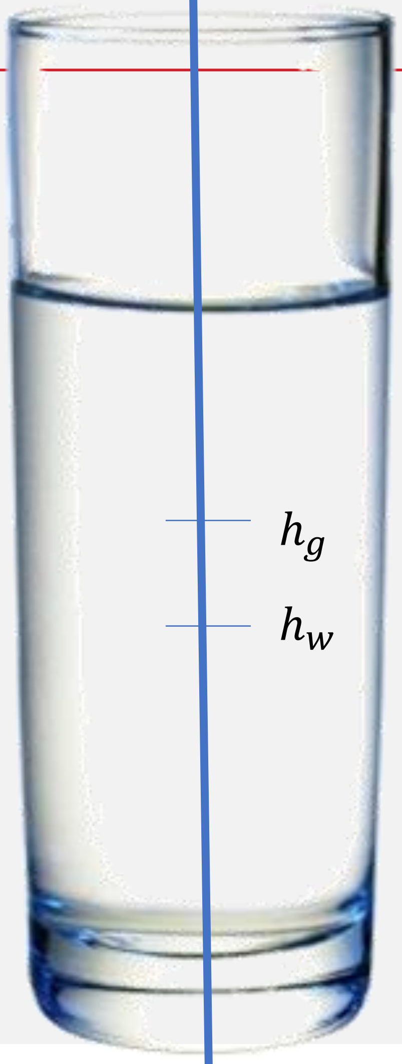
The key concept now is center of gravity!

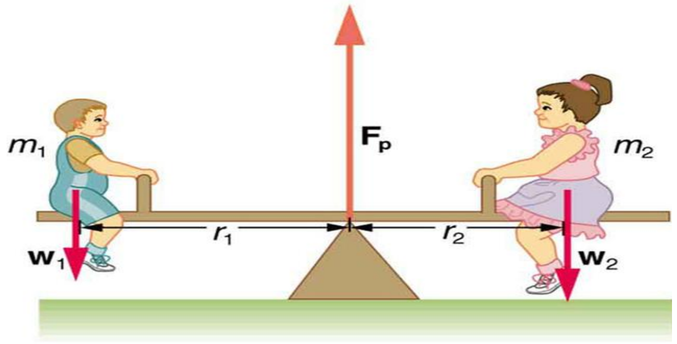
- The **lowest** the center of gravity, the **highest** the stability.
- Let us examine the glass and ignore the water for now.
- It's a cylinder whose walls are homogeneous.
- Given the symmetry of its base, the center of gravity is somewhere along the vertical line through the center of the base.
- If (for **simplicity**) we ignore the mass of the bottom then its center of gravity is halfway its height, at height h_g.



With respect to the water…

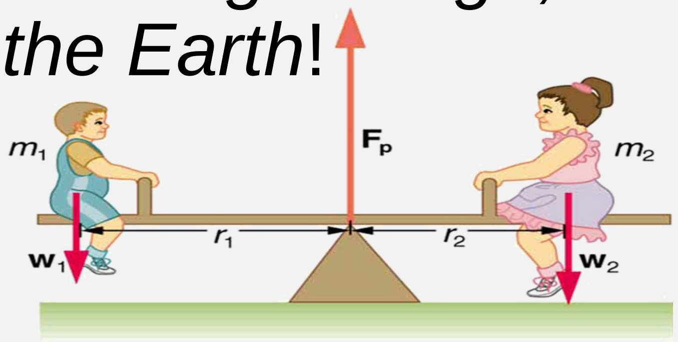
- The water inside the glass also takes the shape of a regular cylinder.
- The center of gravity of the water is also on the vertical line through the center, at halfway the height of the water, at height h_w.
- Now we need to determine the height h of the center of gravity of the **glass with water**.




m_1
$\mathbf{F_p}$
m_2
r_1
r_2
$\mathbf{w}_1$
$\mathbf{w}_2$

Archimedes van Syracuse, Magna Graecia 287-212bc

- Taught us many things, including how to balance mass using levers.
- *Give me a fulcrum and a lever that is long enough, and I can lift the Earth*!

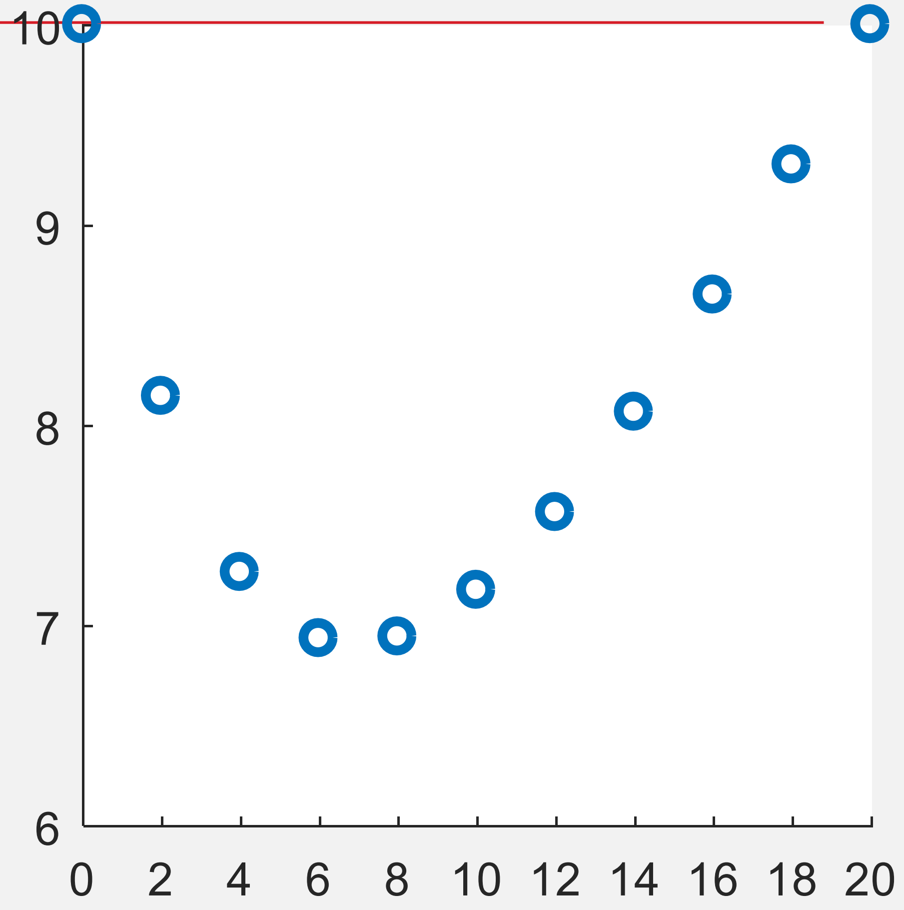


$$m_1 r_1 = m_2 r_2$$

Center of gravity as a function of the amount of water

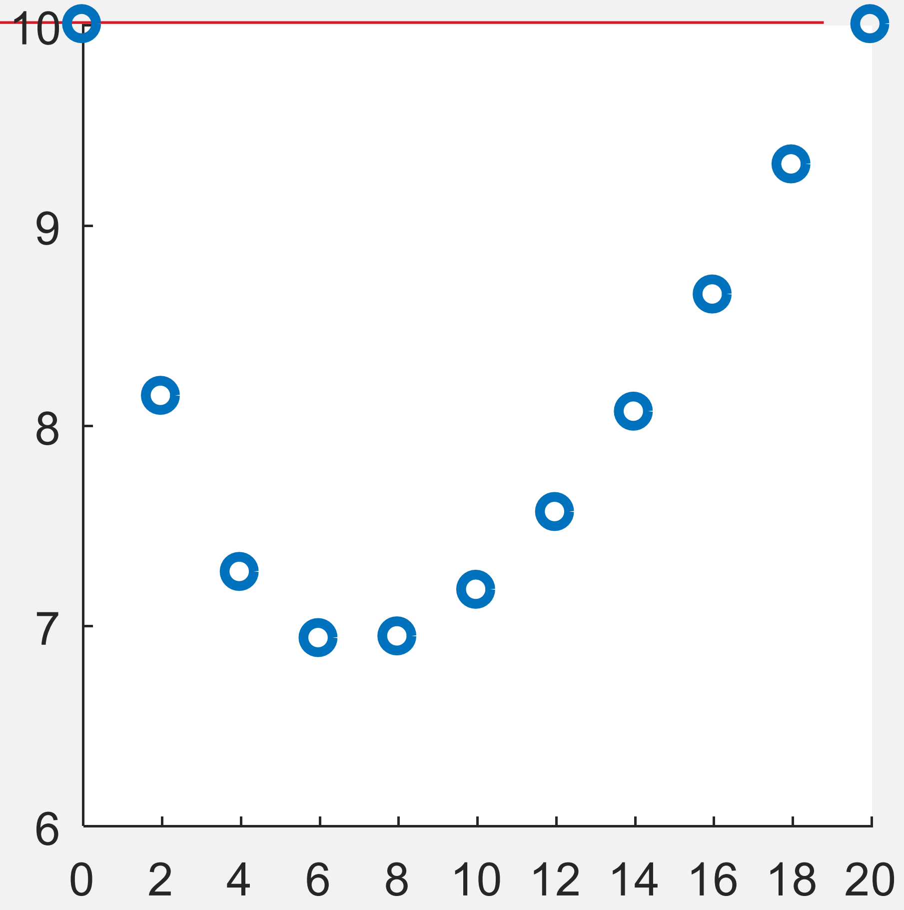
- The center of gravity of the glass with water will be **between** two centers of gravity:
  - that of the empty glass
  - and that of the water alone.
- The height will be in **between** the height of the two individual centers of gravity.
- The **exact** height h respects the Law of the Lever as taught by Archimedes and is given by:
  - $h = \frac{m_w}{m_w+m_g}\, h_w + \frac{m_g}{m_w+m_g}\, h_g$
- with m_w the mass of water and m_g the mass of glass.

Suppose that our glass is 20 cm tall, 4 cm wide and weighs 100 gram



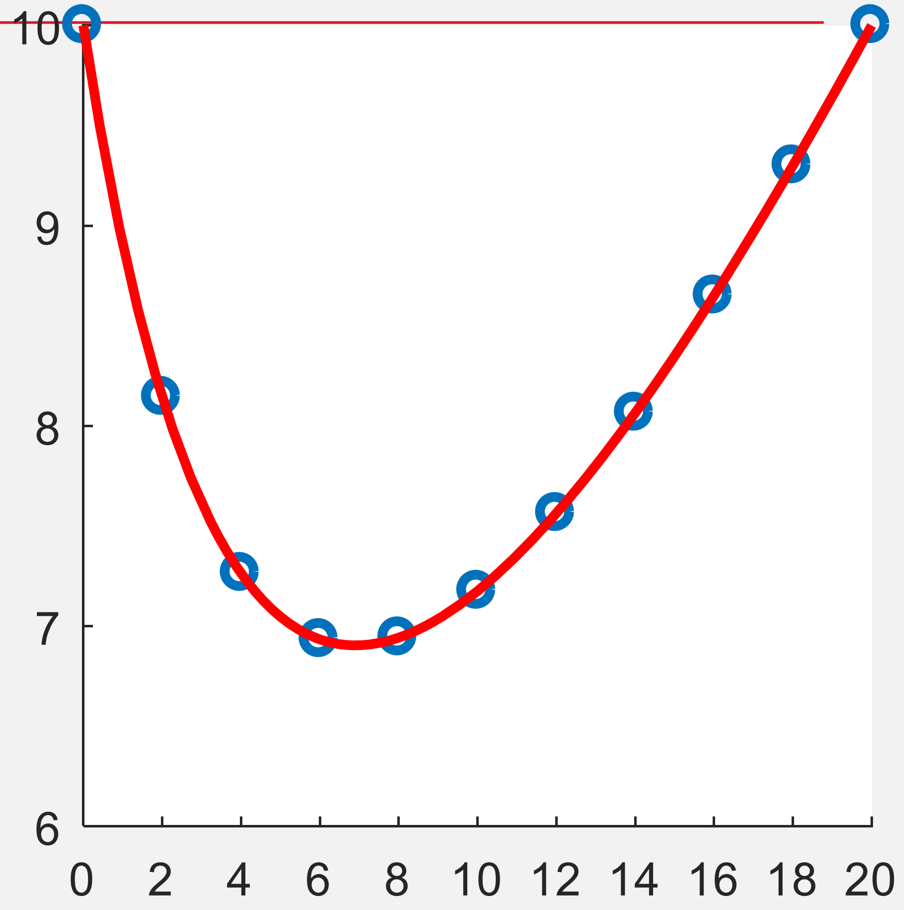
Suppose that our glass is 20 cm tall, 4 cm wide and weighs 100 gram

- Alice could fill the glass with water up to height x cm, provided $0 \leq x \leq 20$ since the water must fit in the glass.
- The volume of water is $\pi r^2 x$ with r the radius of the base: $r = 2$. The volume is therefore approximately $13x \text{ cm}^2$.
- Since the mass of 1 cm^2 of water equals 1 gram we can finally list all relevant values:
  - $h_g = 10$ and $m_g = 100$
  - $h_w = \frac{x}{2}$ and $m_g = 13x$
  - $h = \frac{13x}{(13x+100)}\frac{x}{2} + \frac{100}{(13x+100)}10 = \frac{2000+13x^2}{26x+200}$



Suppose that our glass is 20 cm tall, 4 cm wide and weighs 100 gram

- Alice realizes that **these** points are part of a **bigger** story!
- For each x, thus not only for those in steps of 2,
$$h = \frac{2000 + 13x^2}{26x + 200}$$
- Alice understands that she can **plot** the center of gravity h of the glass with water as a **function** of the height x of the water in the glass.
- She can now **see** that the lowest center of gravity (highest stability) is achieved with about one third of the glass filled with water.
- How could Alice **spot** (find **and** recognize) that optimum?



Pierre de Fermat,
France 1601-1665

- The very **first** person to notice that the derivatives vanish on (unconstrained) extreme points: $f'(x) = 0$ or $\nabla f(x) = 0$.
- He is therefore considered by many to be the **founding father** of *mathematical optimization*.
- Not bad, for a lawyer! And trust me, het left us much more!

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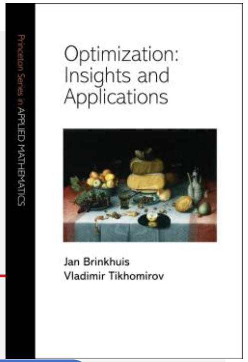
A romantic story reaches a happy end thanks to Archimedes and Fermat!

- $f'(x) = \frac{13\left(13\,x^2 + 200\,x - 2000\right)}{2\,(100 + 13\,x)^2}$
- $f'(x) = 0$ for $x = \frac{60\sqrt{10}}{13} - \frac{100}{13} \approx \frac{20}{3}$
- Fermat taught Alice that **maxima** and **minima** vanish the derivative: the second derivative classifies them.
- $f''(x) = \frac{468000}{(100 + 13\,x)^3}$ which is positive on $\frac{20}{3}$ indicating that this is a minimum.
- Now Alice fills, **full of confidence**, one third of the glass and places the rose in the most stable possible vase!
- After this exciting story with a happy ending, time has come to introduce ourselves.

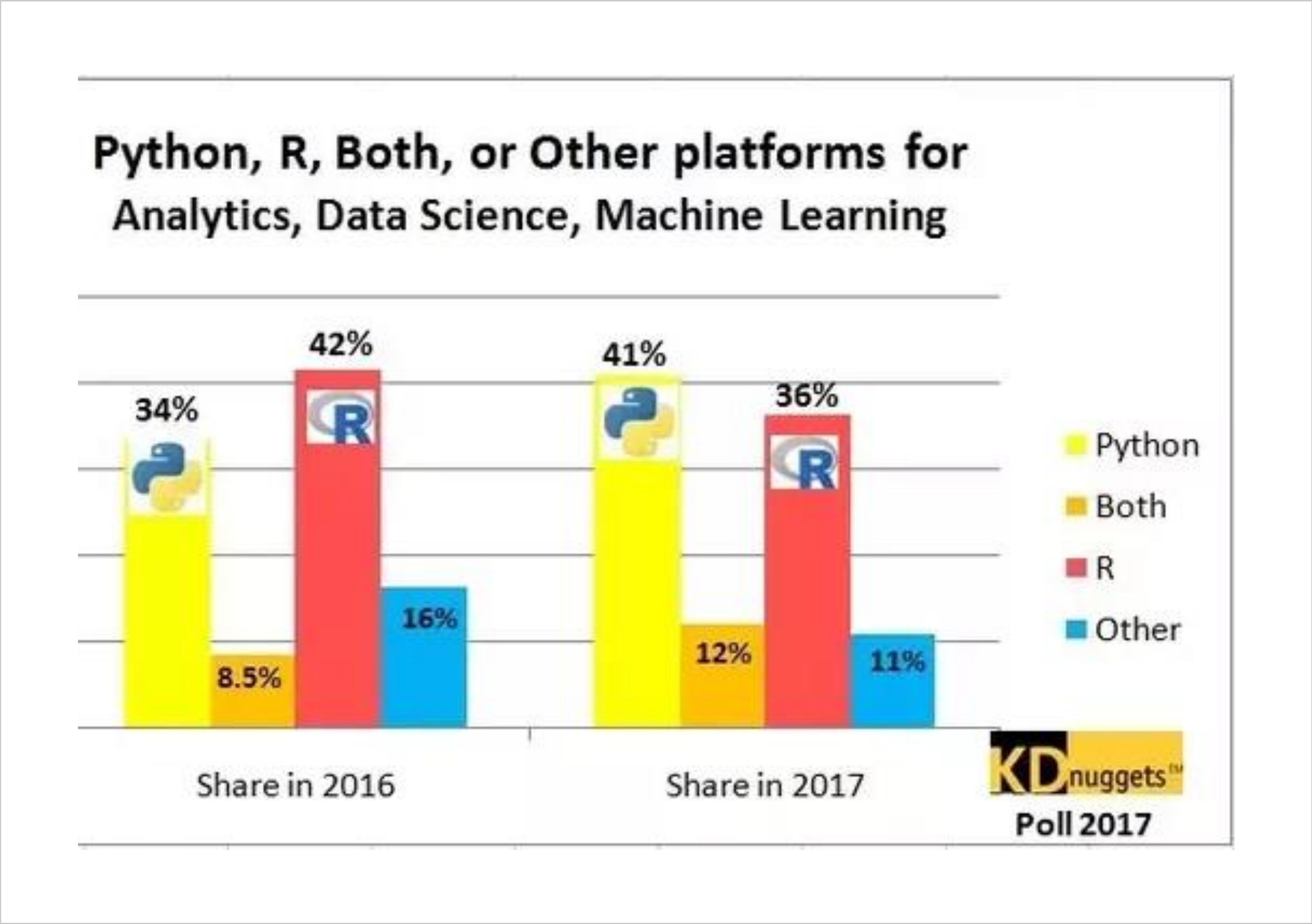
What have we learned?



- That a mathematical optimization model consists of:
  - **Decision variables**.
  - An **objective function**.
  - **Functional constraints**.
  - An **aim** for the objective.
- That mathematicians have found ways to **describe** optimal solutions. So far, without active (or binding) constraints.
- That it may be possible to **solve** mathematical optimization problems **analytically**.
- That **nothing** beats an optimum!

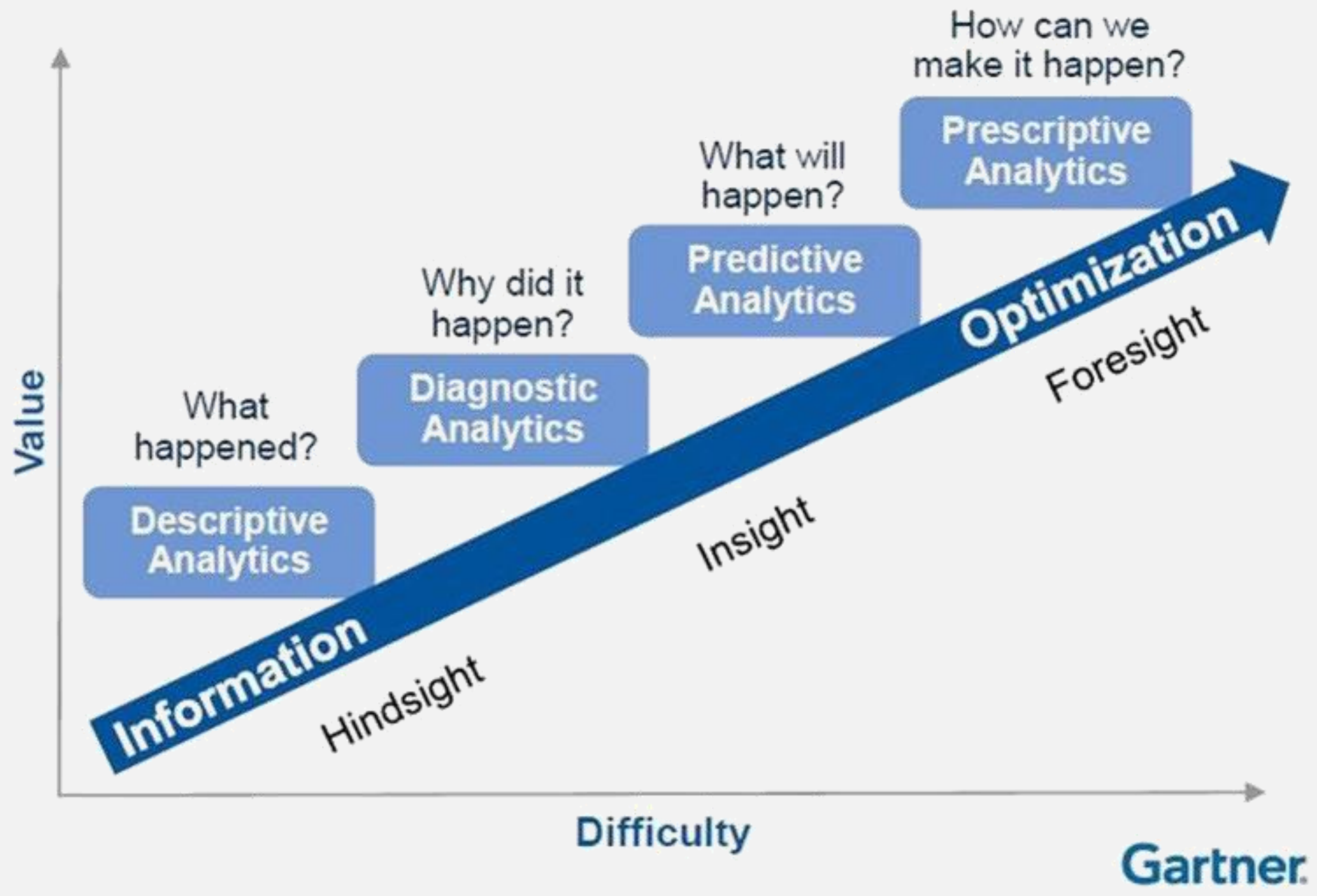
Let us start to put our hands-on optimization…

Did you ever ask yourself this question?



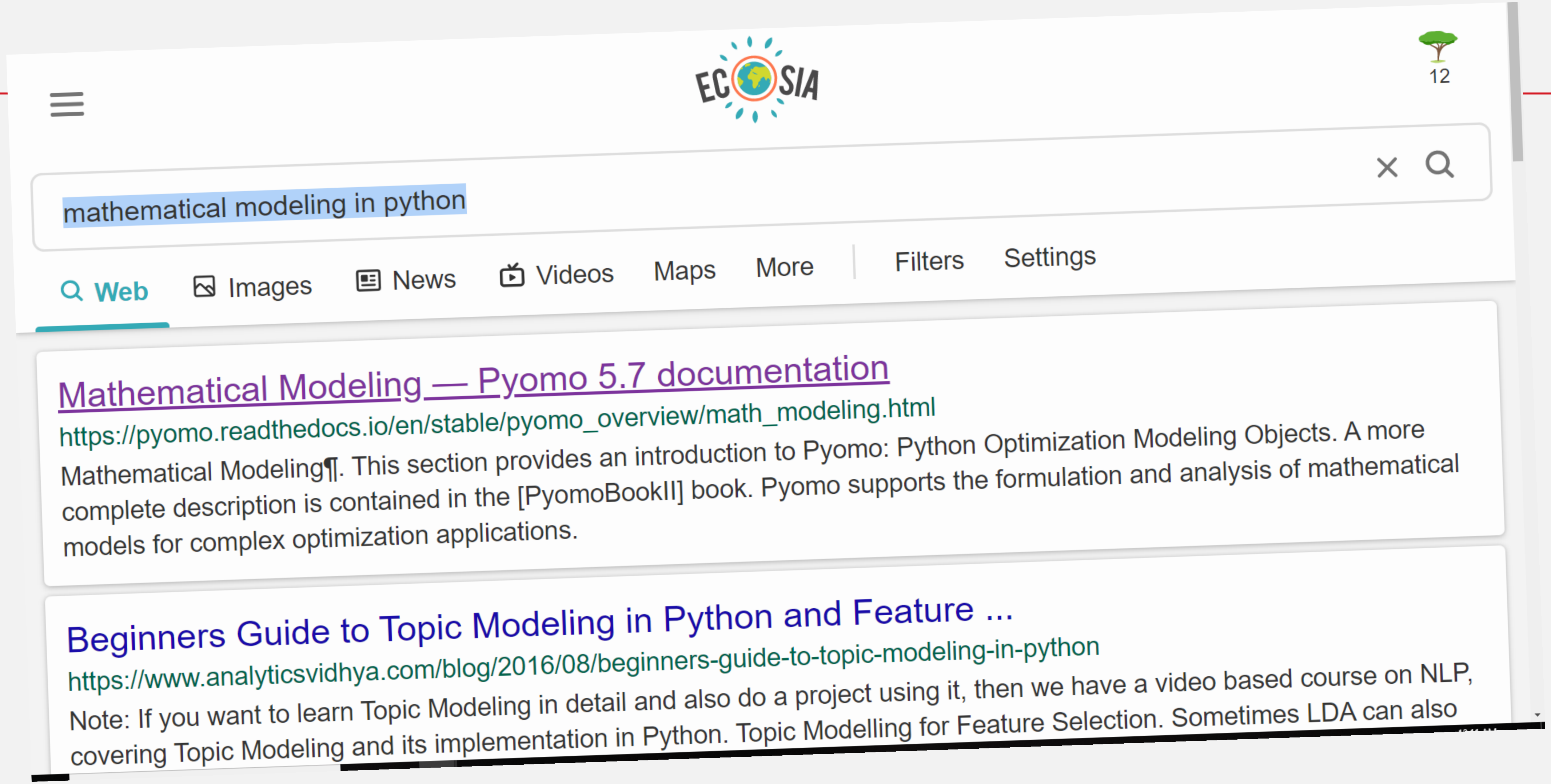
There seem to be two major camps in modern analytics

Given the Gartner four phases of analytics



How I believe the usage of languages is spread

python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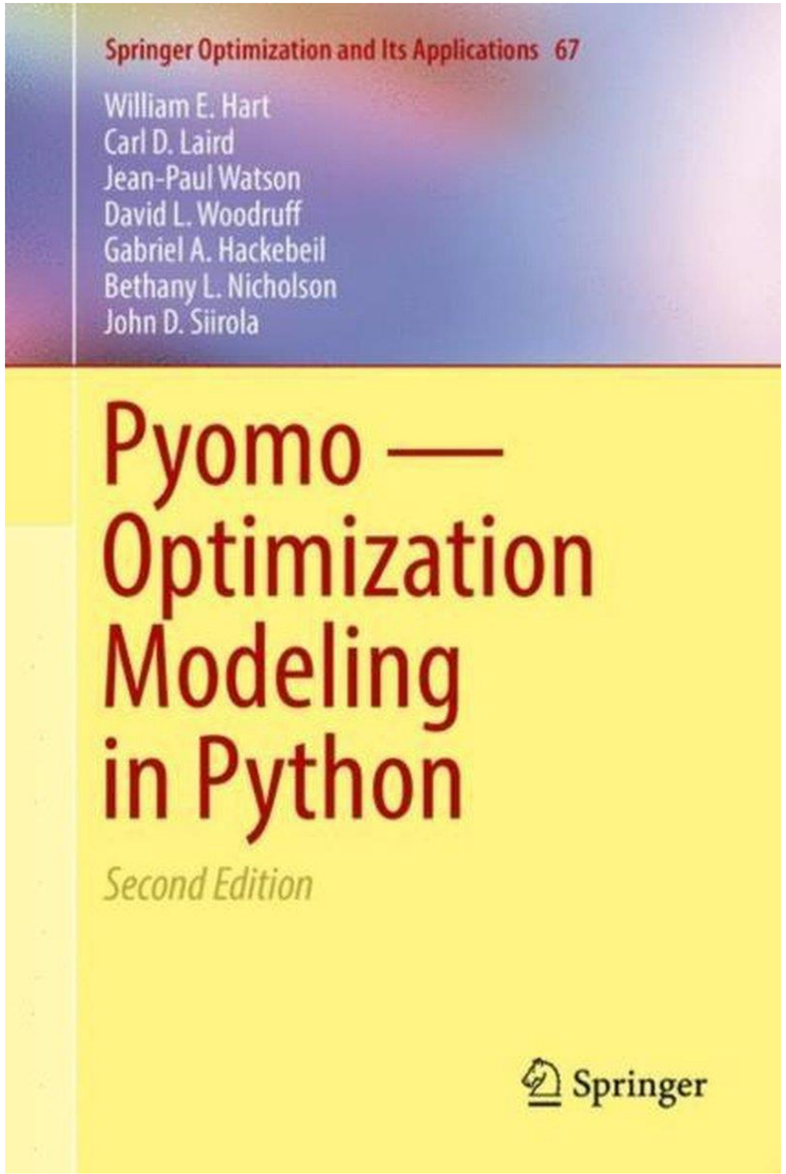
Mathematical Modeling — Pyomo 5.7 documentation
https://pyomo.readthedocs.io/en/stable/pyomo_overview/math_modeling.html
Mathematical Modeling¶. This section provides an introduction to Pyomo: Python Optimization Modeling Objects. A more complete description is contained in the [PyomoBookII] book. Pyomo supports the formulation and analysis of mathematical models for complex optimization applications.
Beginners Guide to Topic Modeling in Python and Feature ...
https://www.analyticsvidhya.com/blog/2016/08/beginners-guide-to-topic-modeling-in-python
Note: If you want to learn Topic Modeling in detail and also do a project using it, then we have a video based course on NLP, covering Topic Modeling and its implementation in Python. Topic Modelling for Feature Selection. Sometimes LDA can also

PYOMO


Springer Optimization and Its Applications 67
William E. Hart
Carl D. Laird
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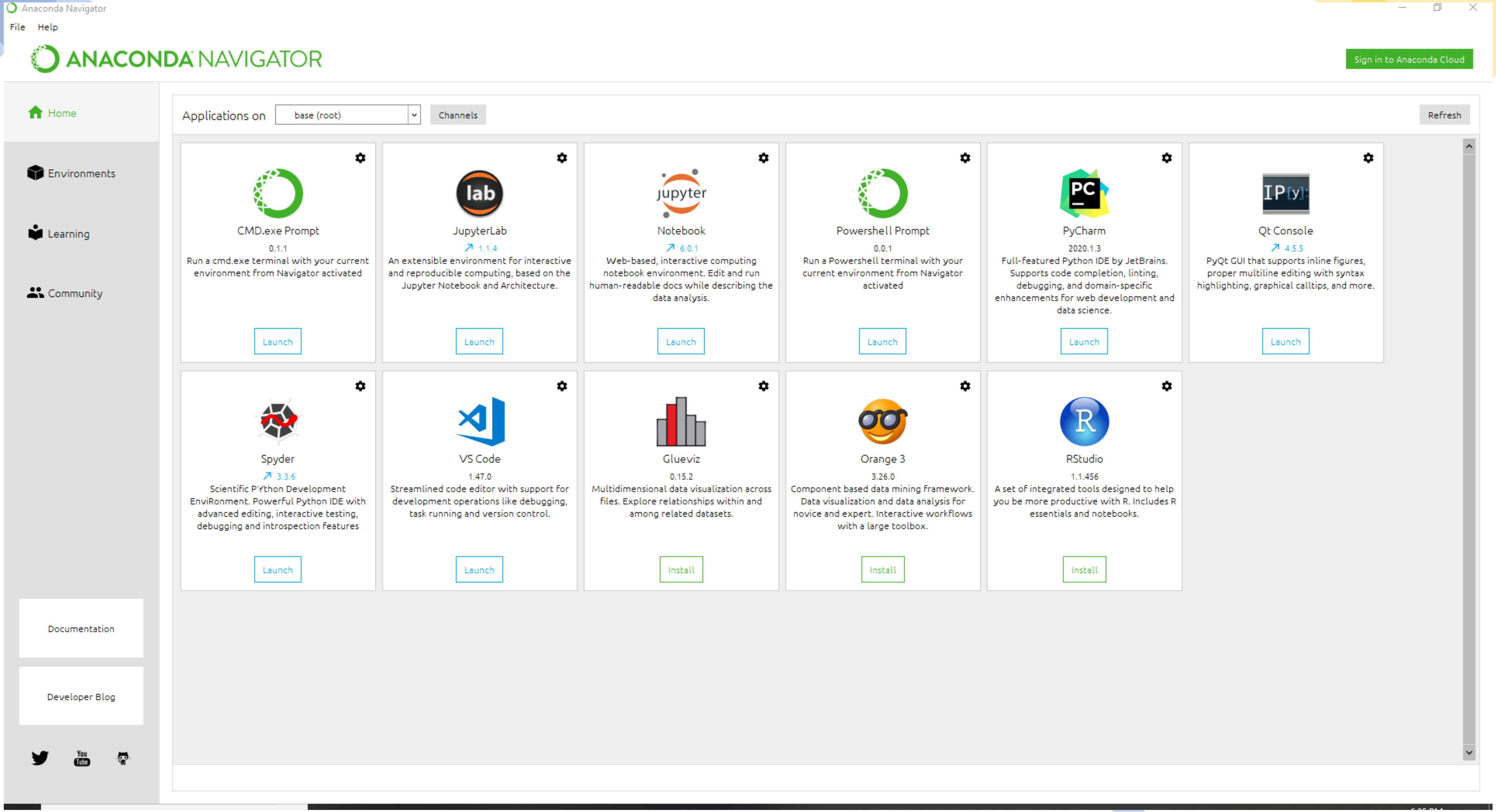
John D. Siirola
Pyomo — Optimization Modeling in Python
Second Edition
Springer



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Run a cmd.exe terminal with your current environment from Navigator activated
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JupyterLab
1.1.4
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Launch
Notebook
6.0.1
Web-based, interactive computing notebook environment. Edit and run human-readable docs while describing the data analysis.
Launch
Powershell Prompt
0.0.1
Run a Powershell terminal with your current environment from Navigator activated
Launch
PyCharm
2020.1.3
Full-featured Python IDE by JetBrains. Supports code completion, linting, debugging, and domain-specific enhancements for web development and data science.
Launch
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4.5.5
PyQt GUI that supports inline figures, proper multiline editing with syntax highlighting, graphical calltips, and more.
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Spyder
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Scientific PYthon Development EnviRonment. Powerful Python IDE with advanced editing, interactive testing, debugging and introspection features
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VS Code
1.47.0
Streamlined code editor with support for development operations like debugging, task running and version control.
Launch
Glueviz
0.15.2
Multidimensional data visualization across files. Explore relationships within and among related datasets.
Install
Orange 3
3.26.0
Component based data mining framework. Data visualization and data analysis for novice and expert. Interactive workflows with a large toolbox.
Install
RStudio
1.1.456
A set of integrated tools designed to help you be more productive with R. Includes R essentials and notebooks.
Install

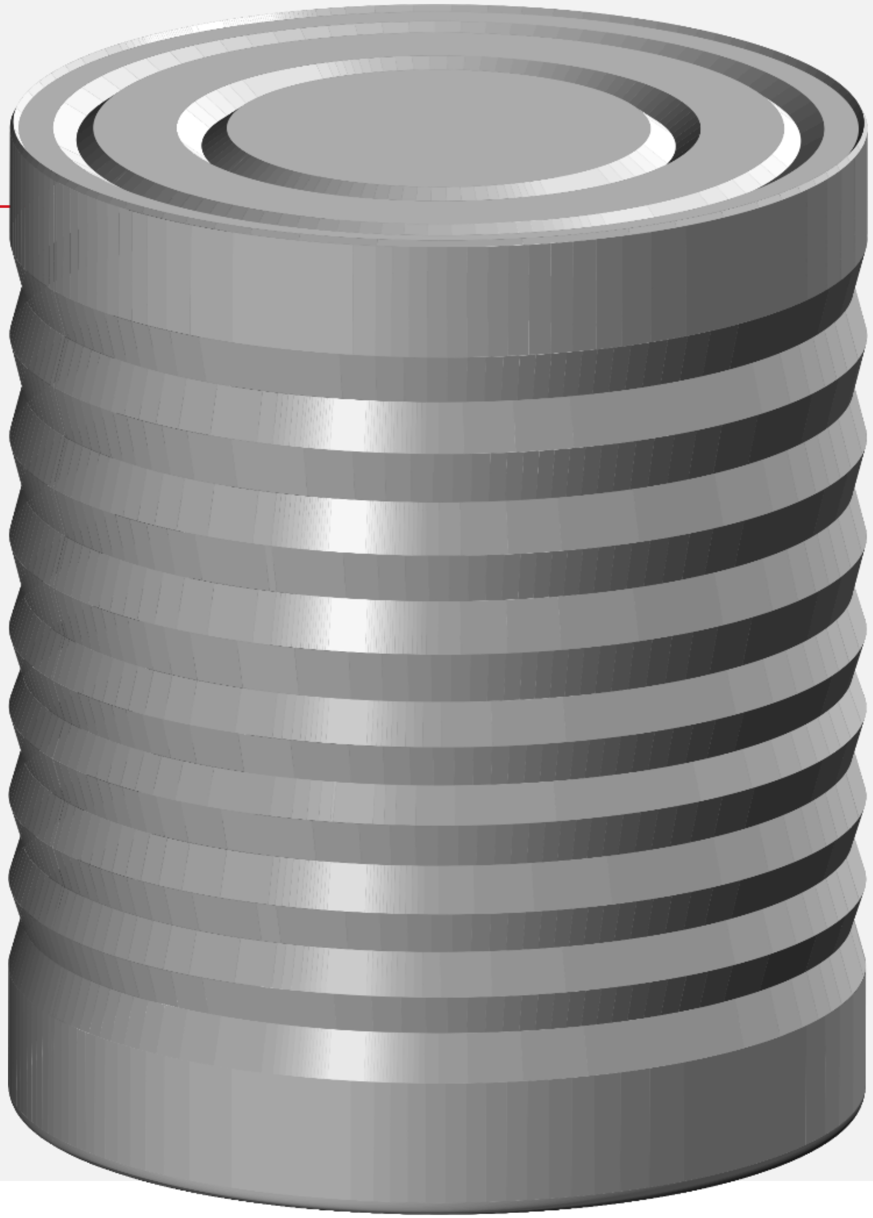
jupyter

This the moment to open the Alice notebook

Meet Betty

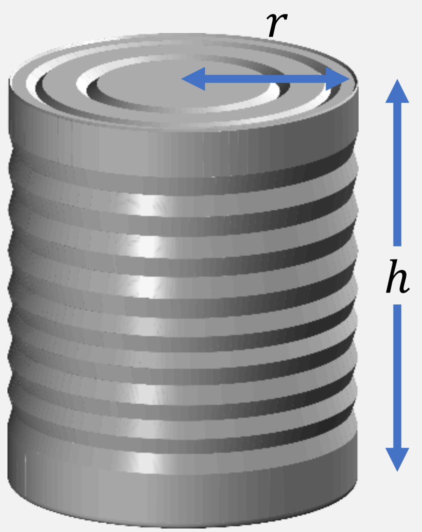
- Betty wants to build a cylinder with a top and a bottom (like a can) which has maximum volume while its weight is limited.

- The weight limitation is equivalent to limiting the surface not to exceed 12 area units.



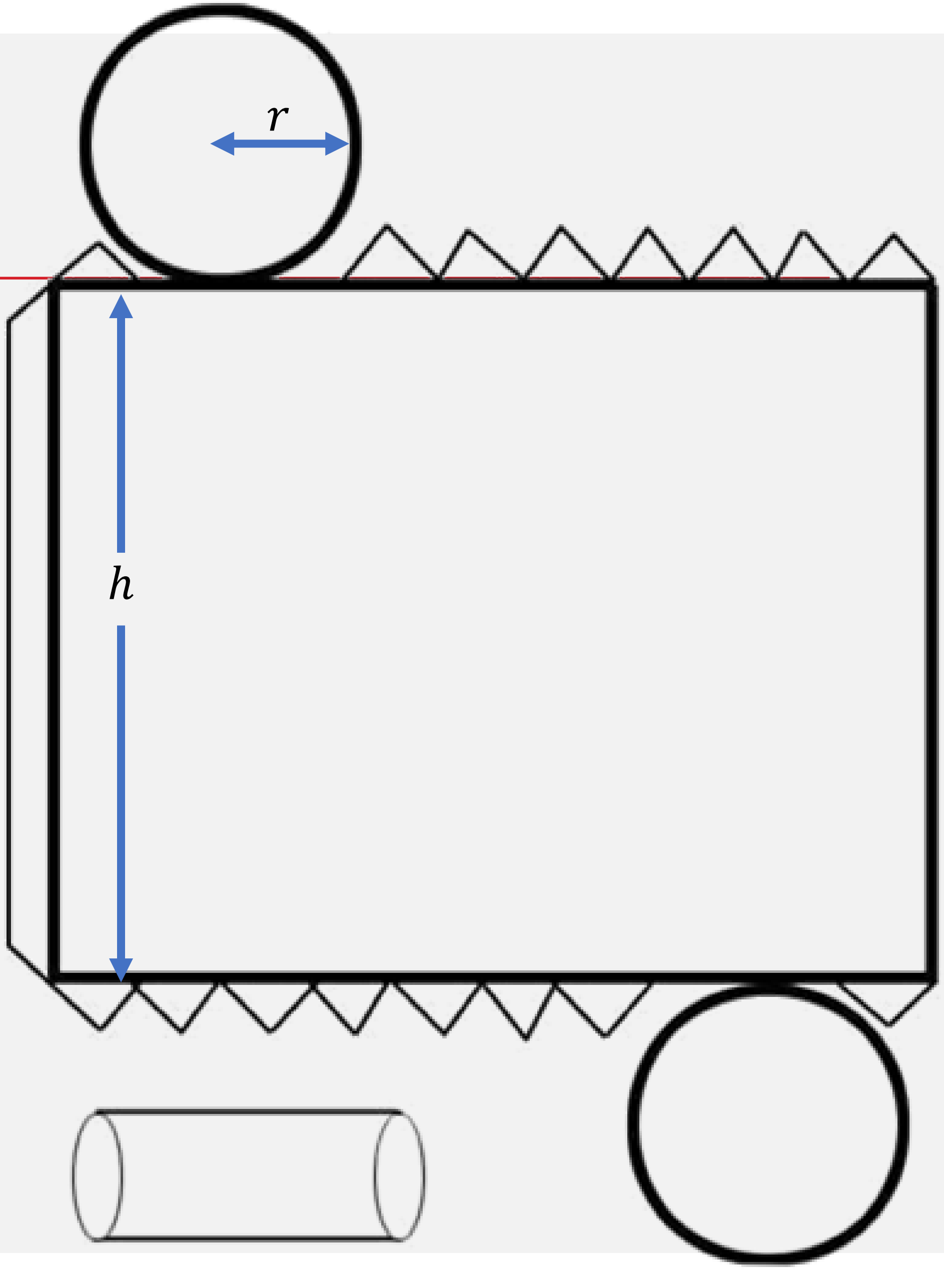
A **new** story!

- Alice had one **decision variable** (height of water) now Betty has two: height and radius of the cylinder.
- Alice had two **restrictions** (above 0 and below 20) and now Betty has three:
  - **Nonnegative** radius r
  - **Nonnegative** height h
  - Surface **not exceeding** 12
- Alice had one **objective** (lower center of gravity), Betty as well: **maximize volume**.
- Some sensible **simplifications**, such as assuming that the walls, basis and top of the cylinder are flat.
- Alice had the lever of Archimedes; Betty knows formulas for the surface and volume.



Filling in the details

- Betty imagines a cylinder being built from flat material.
- If she **prints** this slide and **cuts** the figure on the right, then she can **assemble** a cylinder!
- Since Betty assembled this prototype cylinder from paper, her construction also includes surfaces to glue. These are not part of the real problem.
- With respect to her problem, Betty needs to consider two identical circles of radius r and a rectangle of h by $2\pi r$ which equals the perimeter of each circle.
- The **surface** is therefore $2\pi r^2 + 2\pi r h$.
- And the **volume** is $\pi r^2 h$, i.e. the surface of the base times the height.



A formal model

$$\text{maximize} \quad \pi r^2 h$$

$$\text{subject to:} \quad 2\pi r^2 + 2\pi r h \leq 12$$

$$r \geq 0$$

$$h \geq 0$$

maximize $\pi r^2 h$

subject to: $2\pi r^2 + 2\pi r h \leq 12$

$r \geq 0$

$h \geq 0$

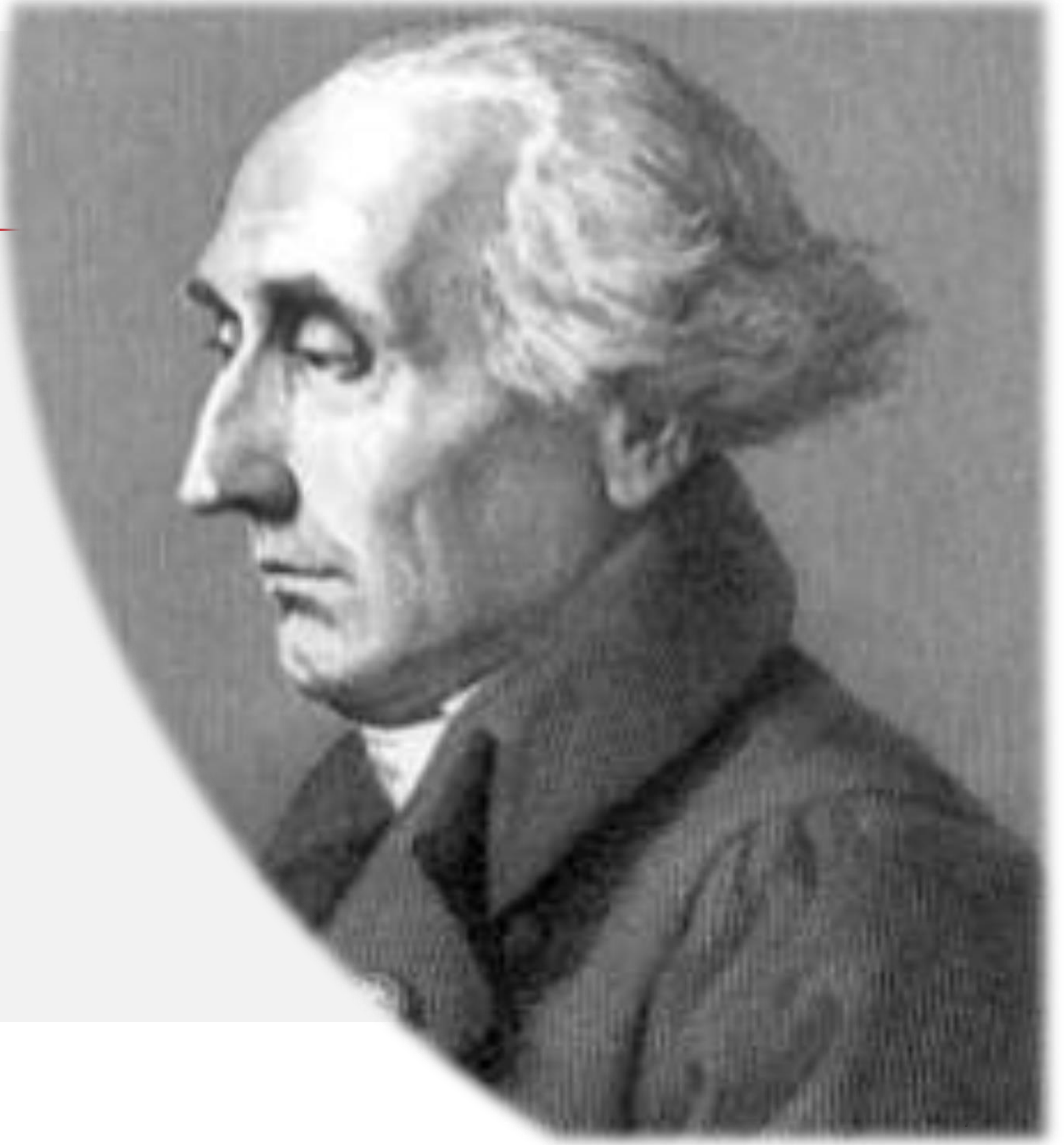
- Intuitively we see that the volume should increase with the surface, therefore we may take the surface constraint as equality: $2\pi r^2 + 2\pi r h = 12$.
- Now we can solve the surface constraint for h and substitute the result into the objective function: $h = \frac{6-\pi r^2}{\pi r}$ leading to the volume being $6r - \pi r^3$.
- We also need to care about the constraints: $h \geq 0 \Rightarrow \frac{6-\pi r^2}{\pi r} \geq 0 \Rightarrow r \leq \sqrt{6\pi}$.
- The problem becomes: maximize $6r - \pi r^3 : 0 \leq r \leq \sqrt{6\pi}$.
- The derivative of the objective function with respect to r is $6 - 3\pi r^2$ which equals 0 for $r = \pm\sqrt{\frac{2}{\pi}}$.
- From the two roots only $r = \sqrt{\frac{2}{\pi}}$ satisfies the constraints. The second derivative is $-6\pi r$ which is negative for all positive radius and hence we found the **maximum**!

And then we arrive at...

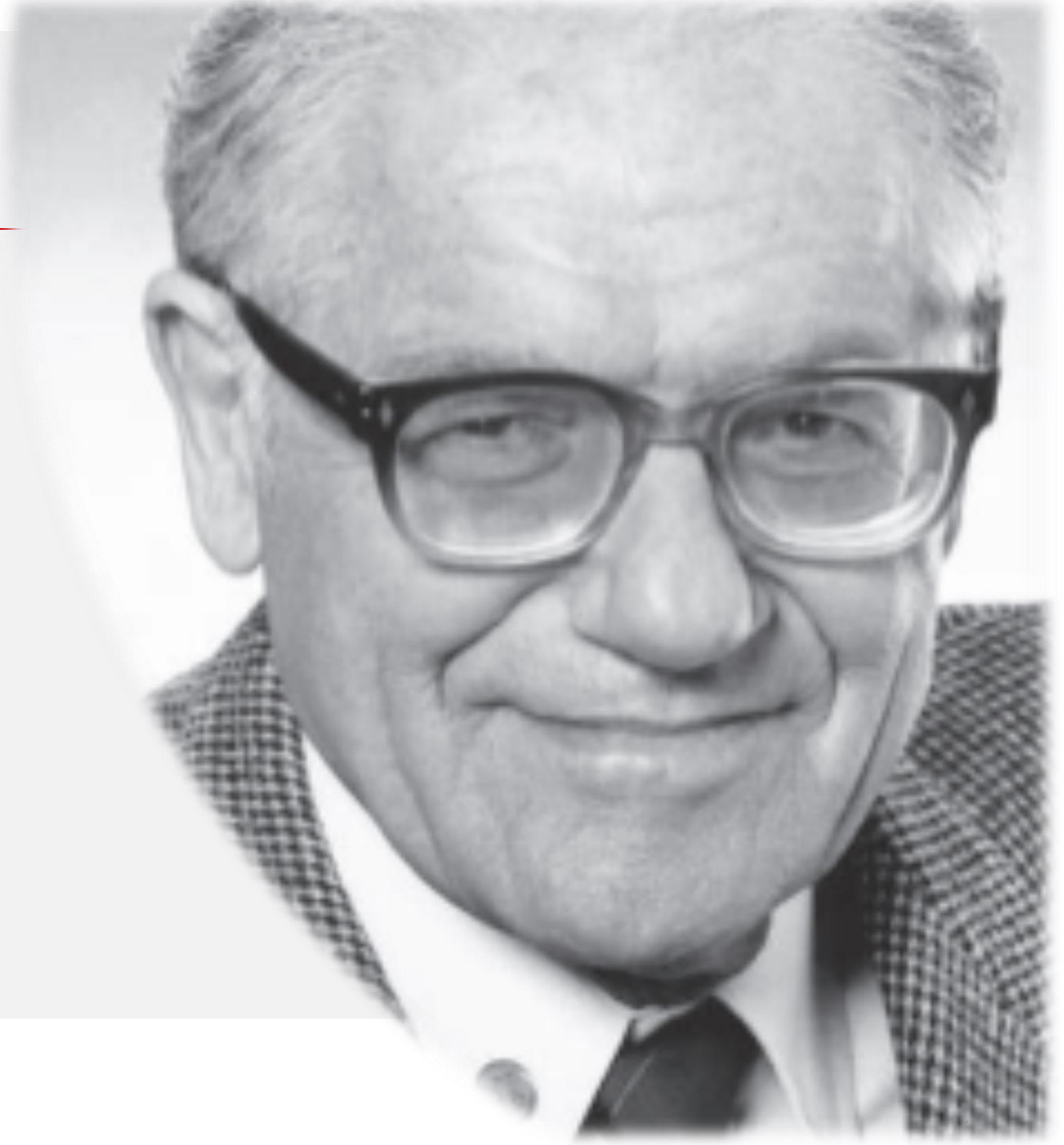
- $r = \sqrt{\frac{2}{\pi}} \approx 0.8$
- $h = \sqrt{\frac{8}{\pi}} \approx 1.6$
- volume = $2\pi\sqrt{\left(\frac{2}{\pi}\right)^3} \approx 3.2$
- surface ≈ 12

Joseph-Louis Lagrange, Italy 1736- France 1813

- Applied Fermat's ideas to the case of functional equality constraints, i.e. $h(x) = 0$.
- In fact, he converted a constrained problem into an unconstrained problem using extra variables, the so-called *Lagrange multipliers*.
- This has led to modern concepts of *duality*.

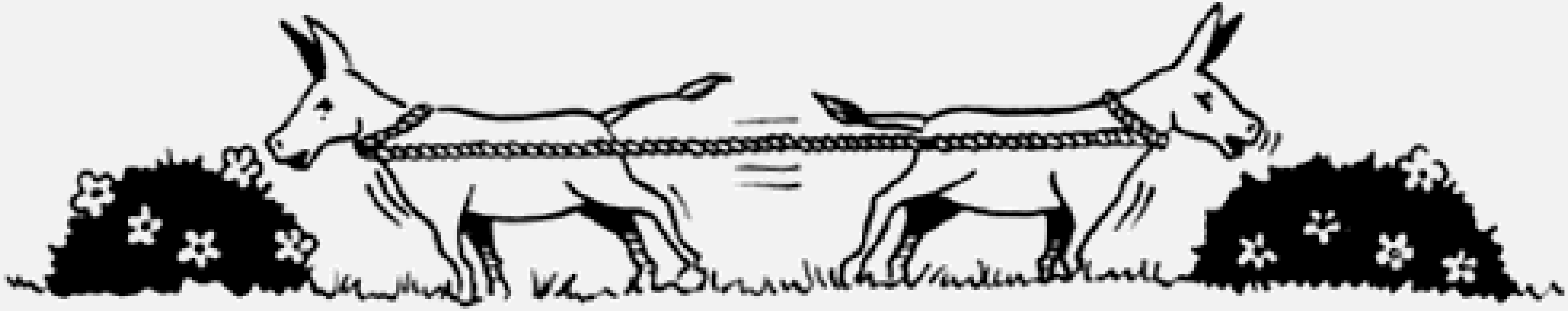


William Karush, USA 1917-1997



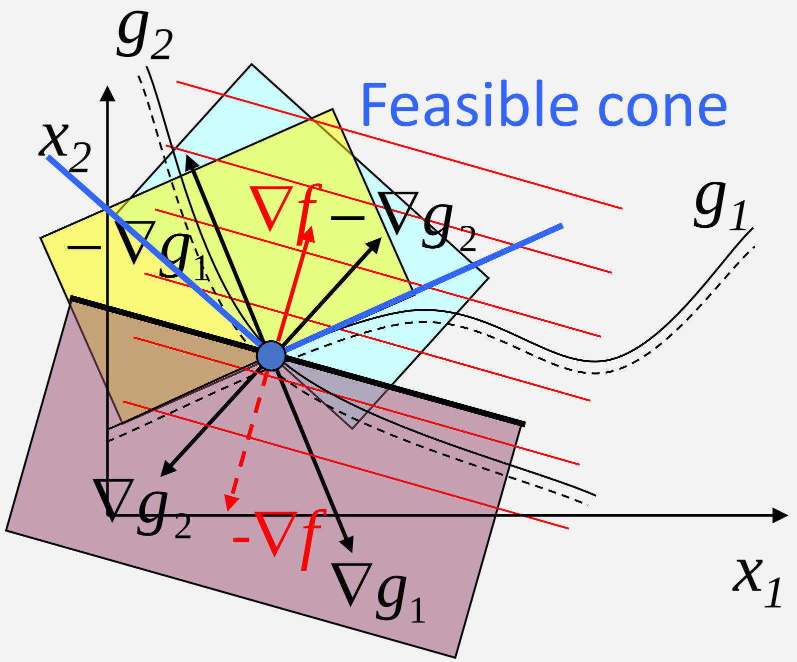
- Extended the work of Lagrange to include *inequalities*, i.e. having constraints of both types
  - $g(x) \leq 0$
  - $h(x) = 0$
- He can be seen as the founding father of *computer based Mathematical Optimization*.
- His findings were part of his **master** thesis and stayed overlooked for some years.
- Later, Professors Harold W. Kuhn and Albert W. Tucker rediscovered Karush's work.
- They were awarded the von Neumann Prize in 1980 for what in fact was first discovered by Karush.
- Now we know these important concepts as Karush-Kuhn-Tucker conditions of optimality.

You prove optimality… by getting stuck!



There may be more ‘forces’ involved

- In case of multiple constraints, each has a saying in terms of determining ‘feasible directions’.
- Geometrically, Karush-Kuhn-Tucker is a result about the intersection of **cones** which are intersections of **half spaces**.
- Only the constraints that are binding have a saying about optimality!

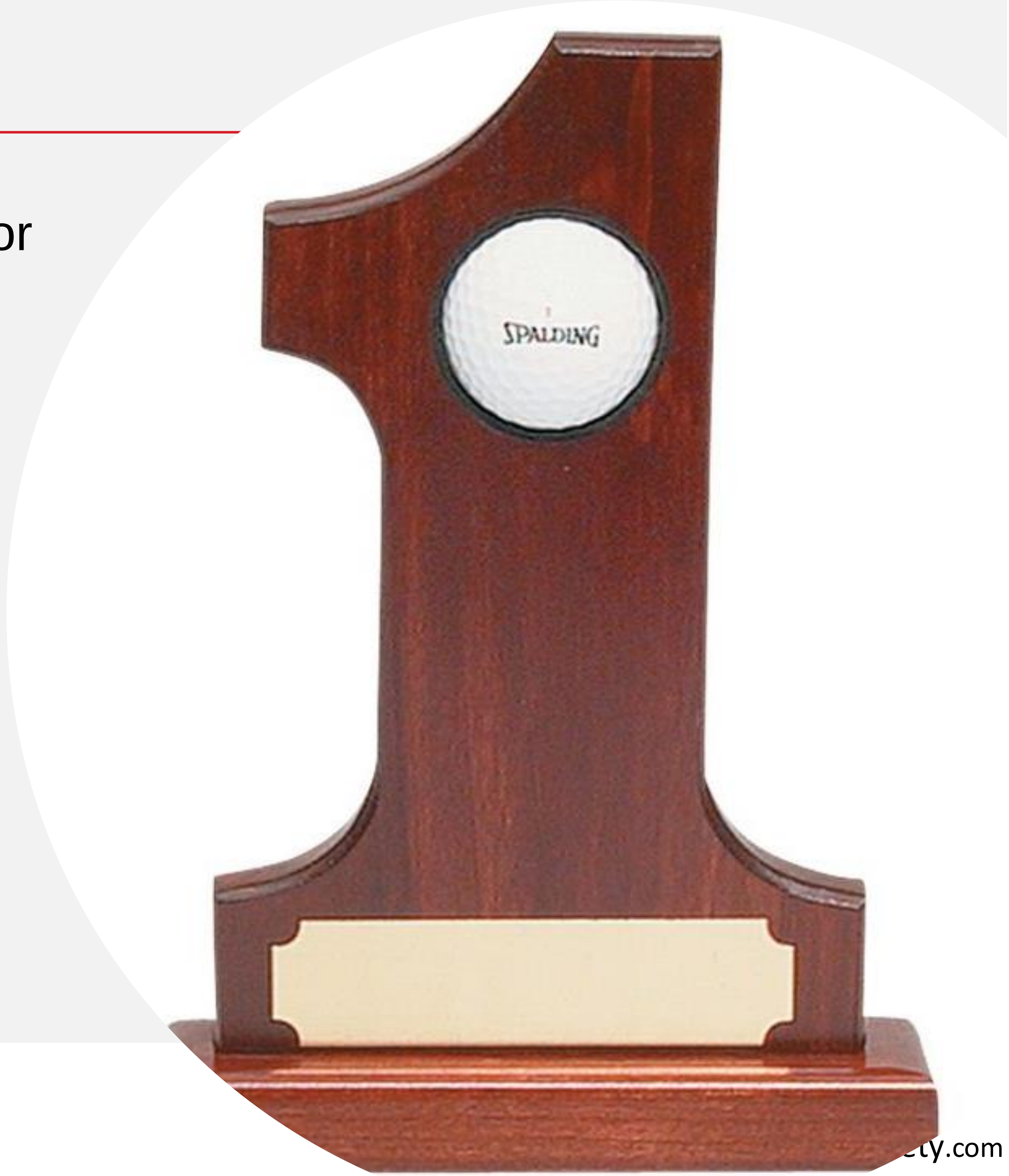


jupyter

This the moment to open the Betty notebook

Caroline’s Trophy Factory

- Caroline owns a company that produces trophies for
  - football
    - wood base, engraved plaque, brass football on top
    - €12 profit and uses 4 dm of wood
  - golf
    - wood base, engraved plaque, golf ball on top
    - €9 profit and uses 2 dm of wood
- Caroline’s current stock of raw materials
  - 1000 footballs
  - 1500 golf balls
  - 1750 plaques
  - 480 m (4800 dm) of wood
- Caroline wonders what the **optimal** production plan should be, in other words: **how many** football and **how many** golf trophies should Caroline produce to **maximize** her profit while **respecting** the availability of raw materials?



Leonid V. Kantorovich, Russia 1912-1986

- Invented linear optimization as a powerful **modeling** instrument in 1939.

- This was kept secret at first but ended up awarding him the **Nobel** prize in economics in 1975.

Start by identifying and **naming** the decisions

- Number of football trophies to produce:

$$x_1$$

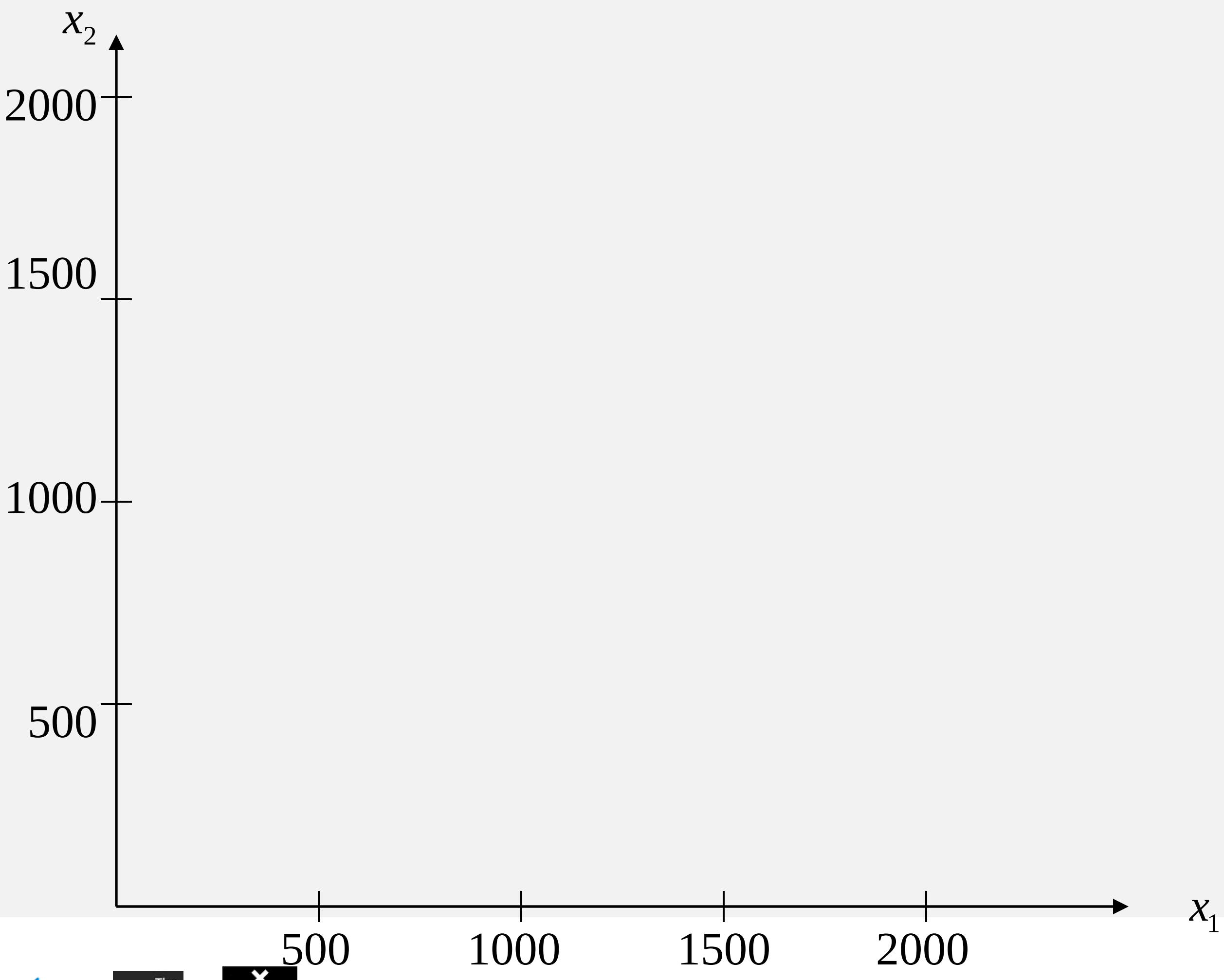
- Number of golf trophies to produce:

$$x_2$$

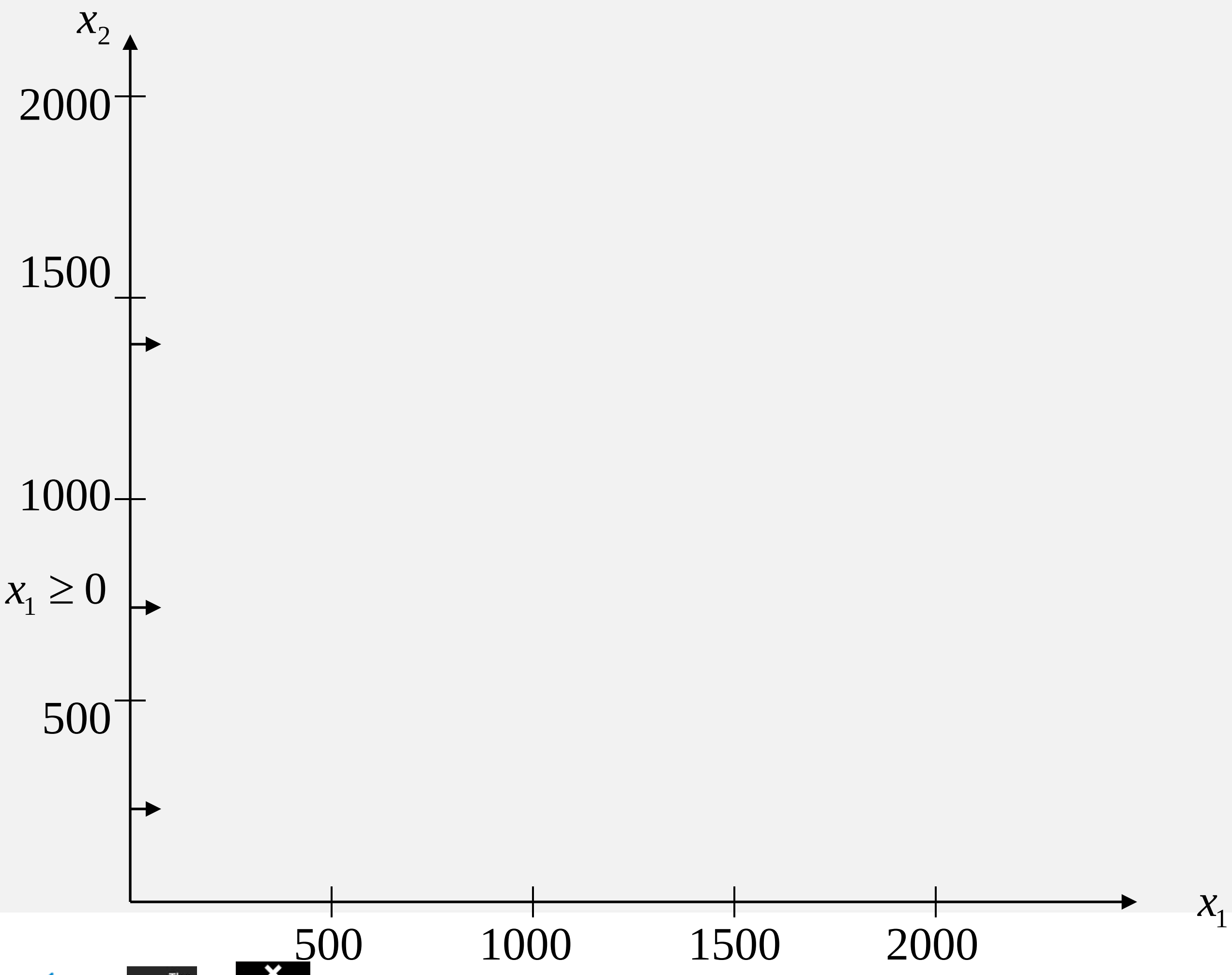
Complete model of Caroline's production optimization

$$
\begin{array}{lrl}
\max & 12x_1 + 9x_2 & \\
\text{s.t.} & x_1 \leq 1000 & \text{Footballs} \\
 & x_2 \leq 1500 & \text{Golf balls} \\
 & x_1 + x_2 \leq 1750 & \text{Plaques} \\
 & 4x_1 + 2x_2 \leq 4800 & \text{Wood} \\
 & x_1, x_2 \geq 0 &
\end{array}
$$

Graphical Solution



Graphical Solution



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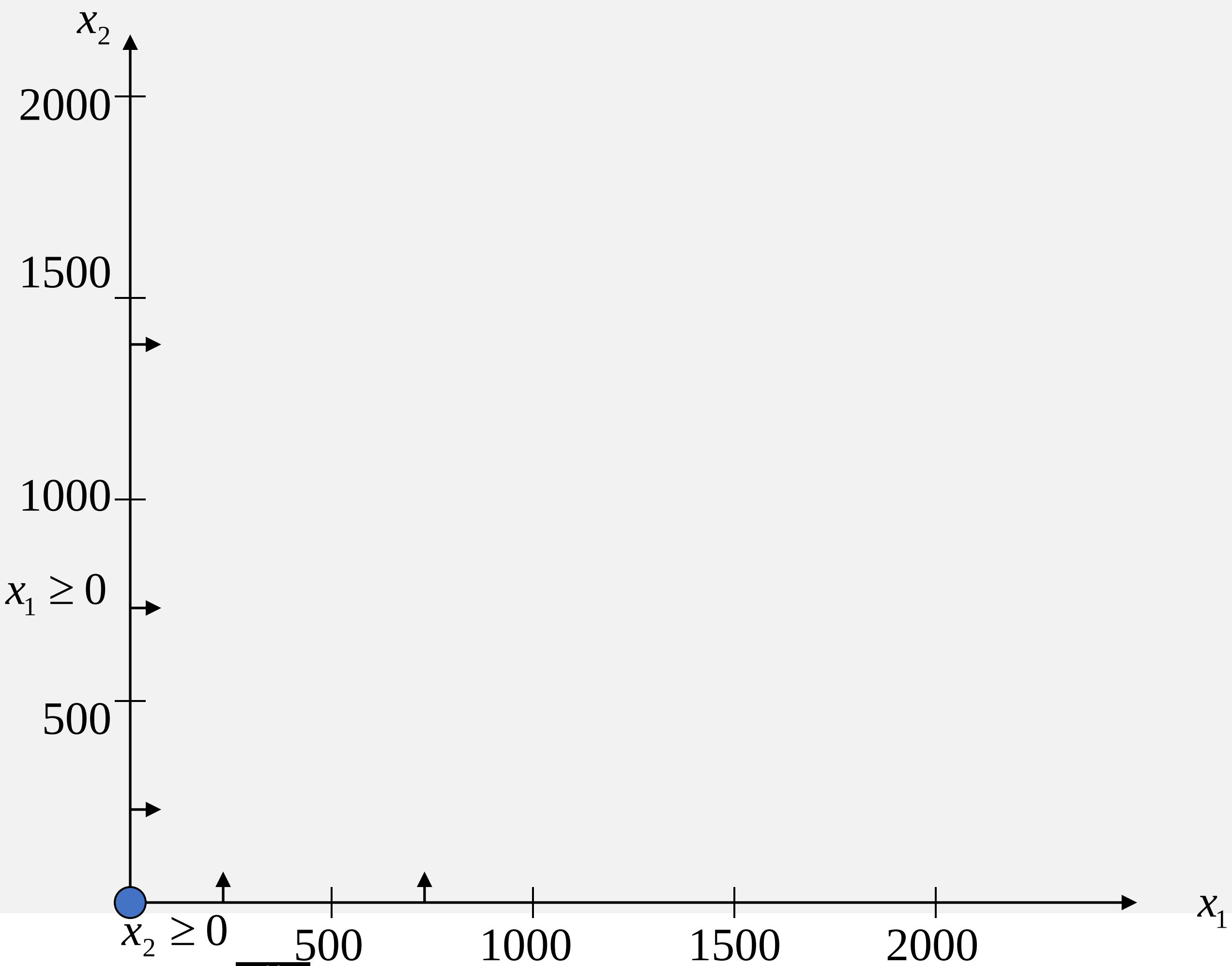
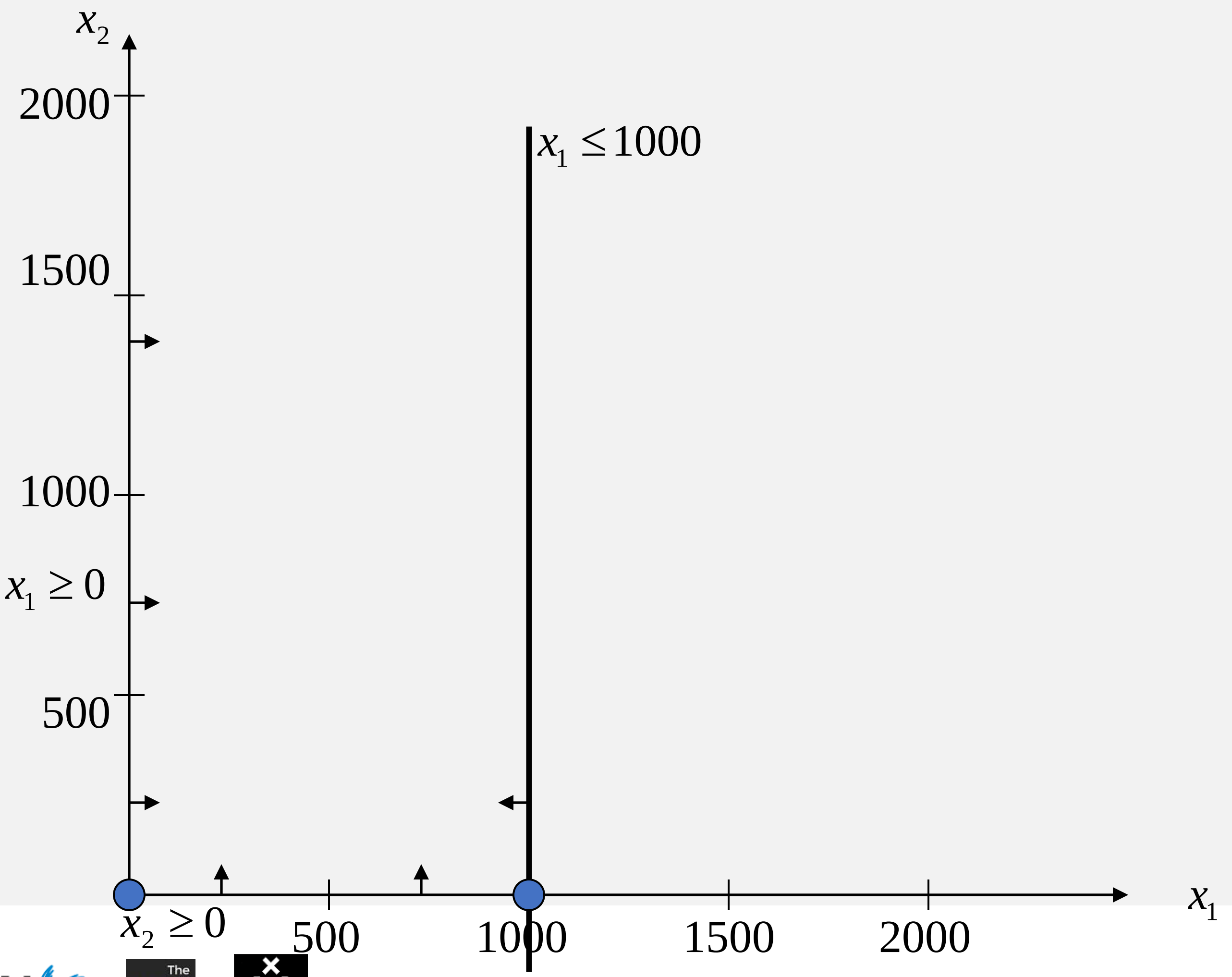
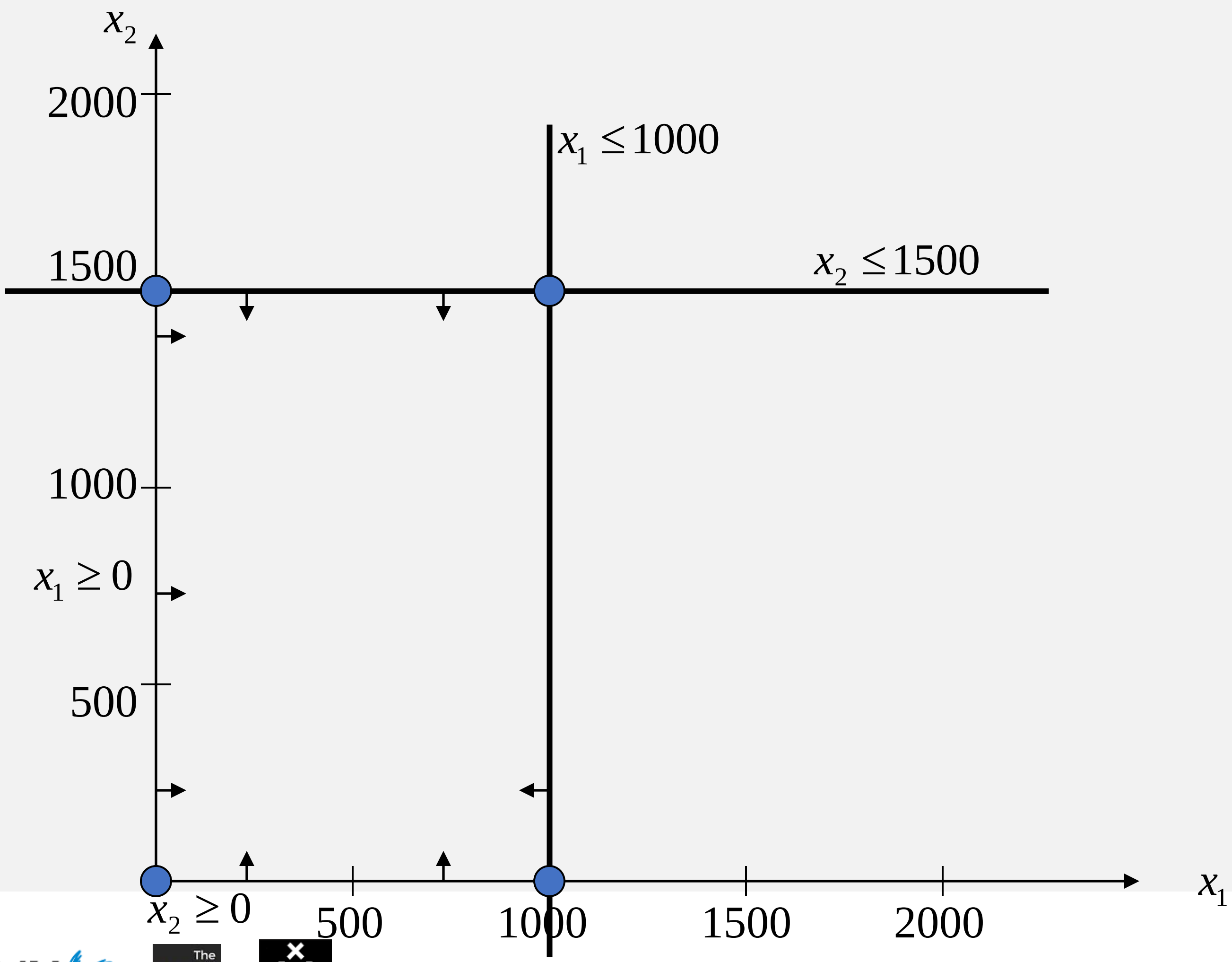


Graphical Solution



Graphical Solution



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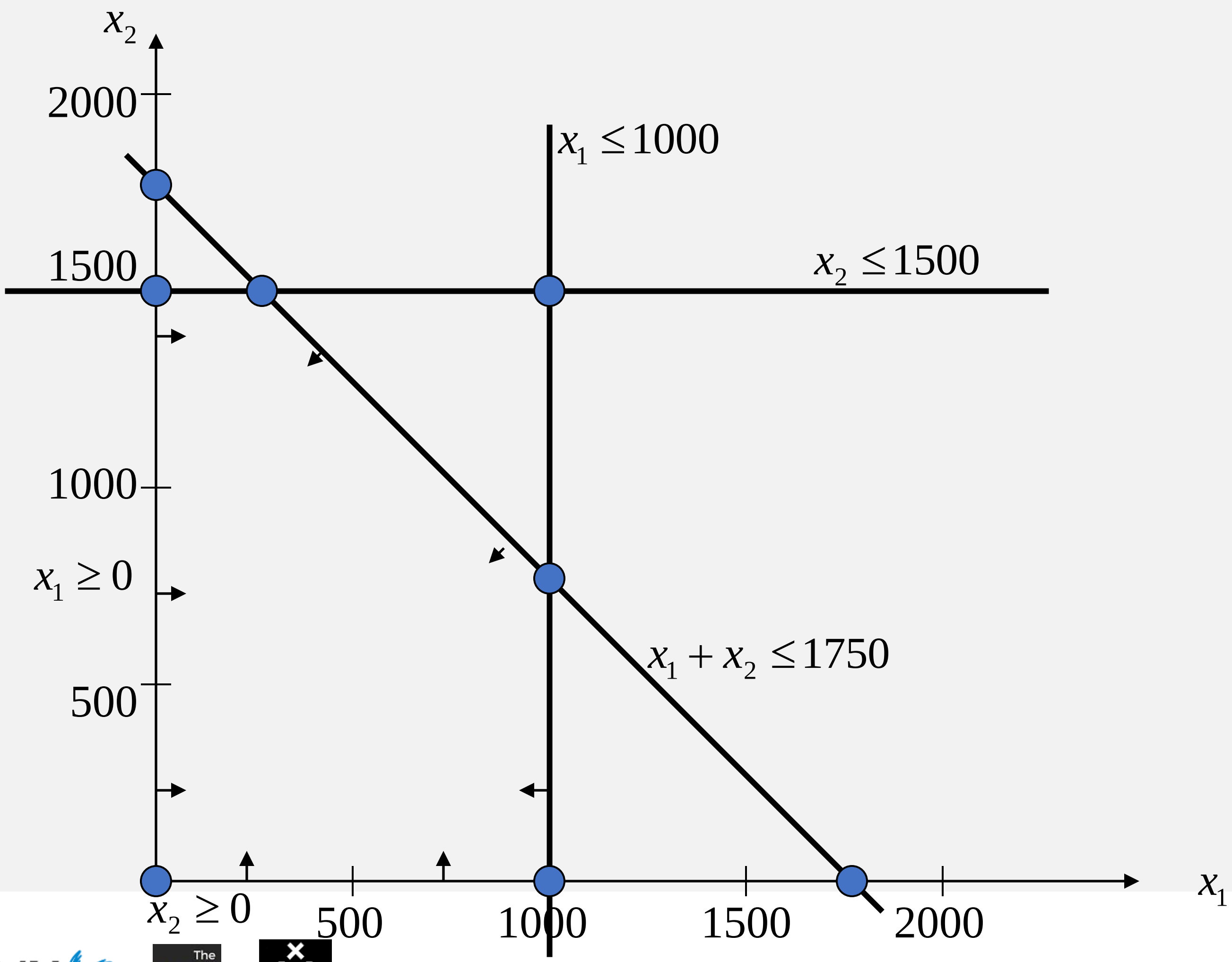

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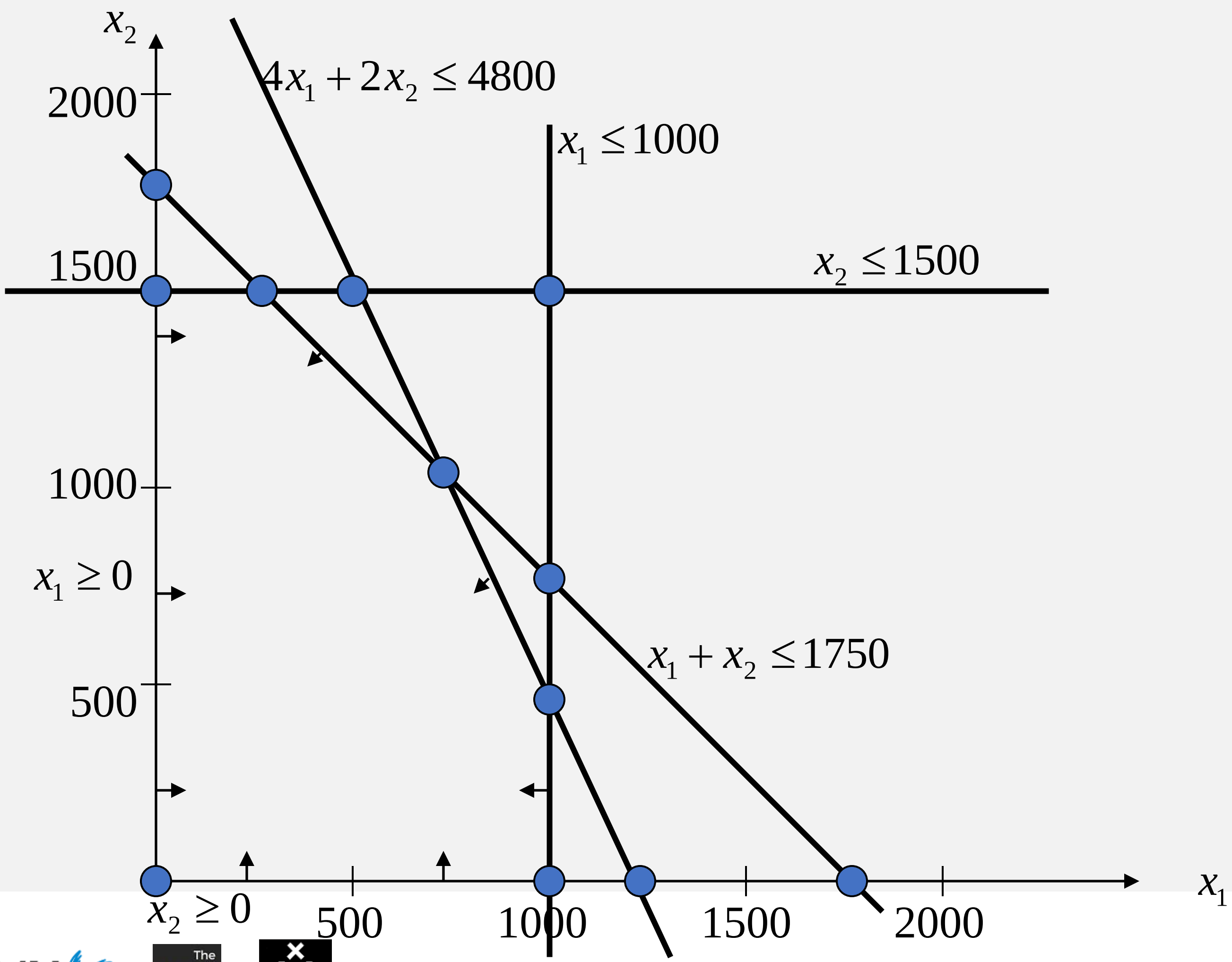

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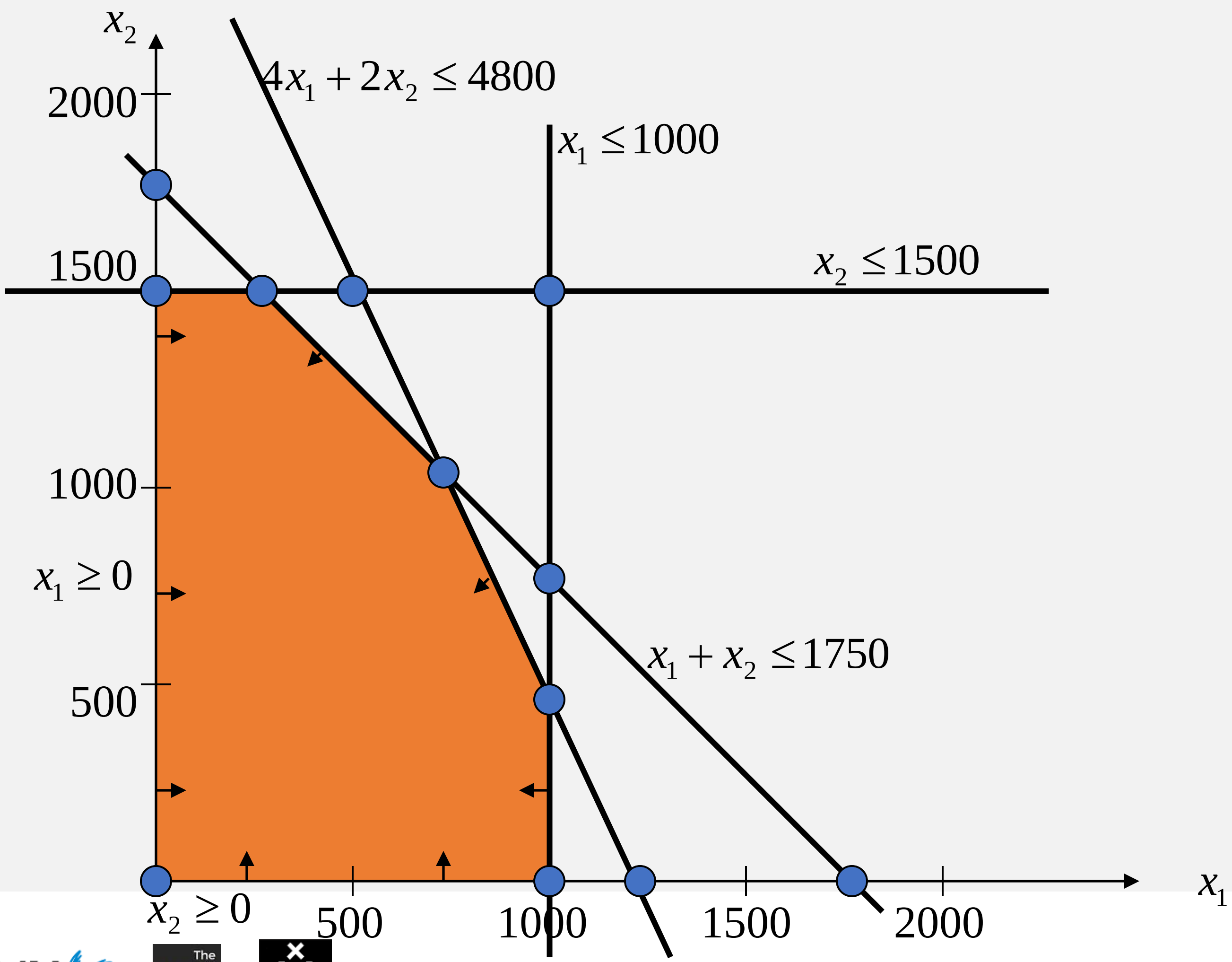

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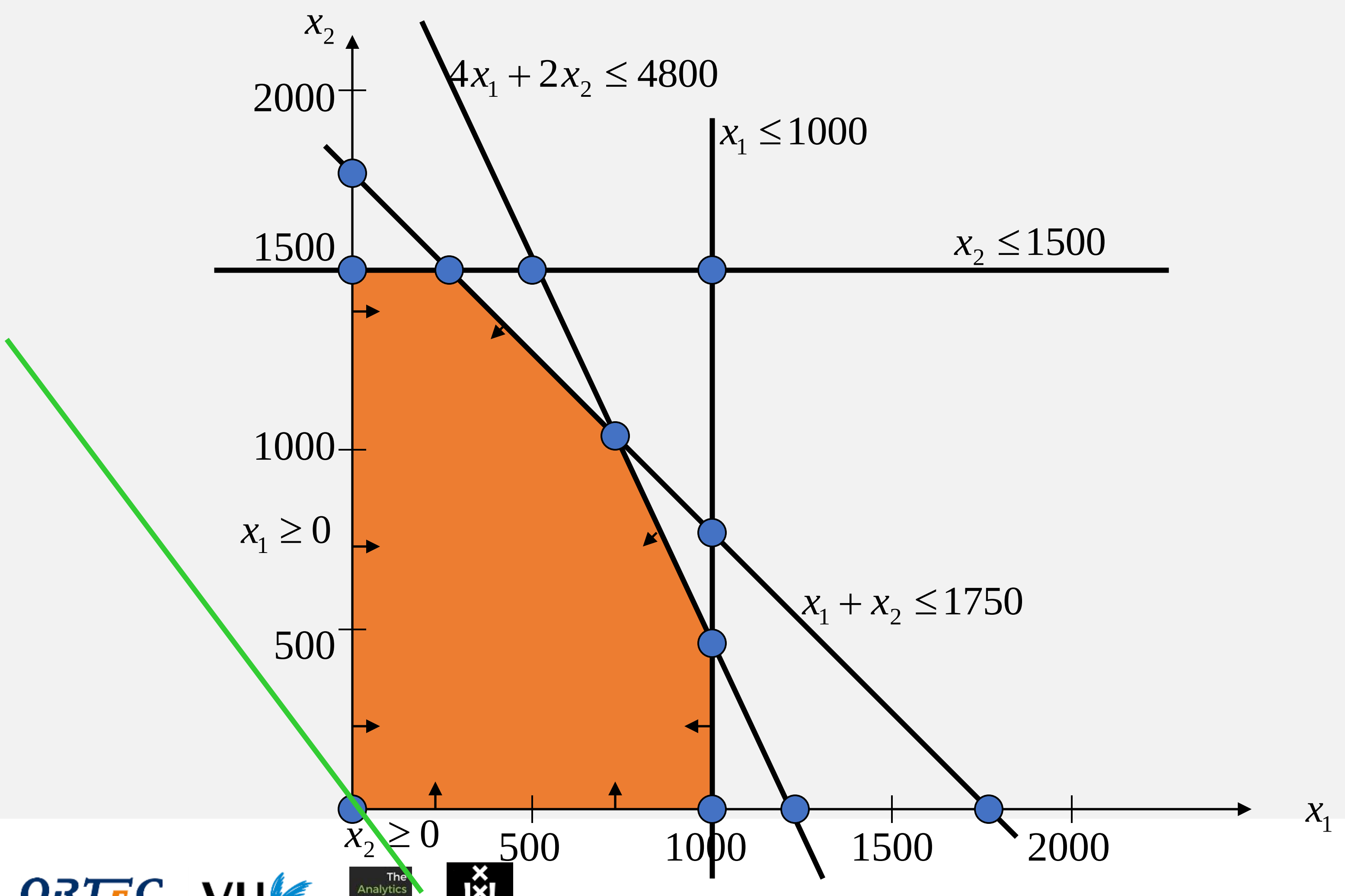

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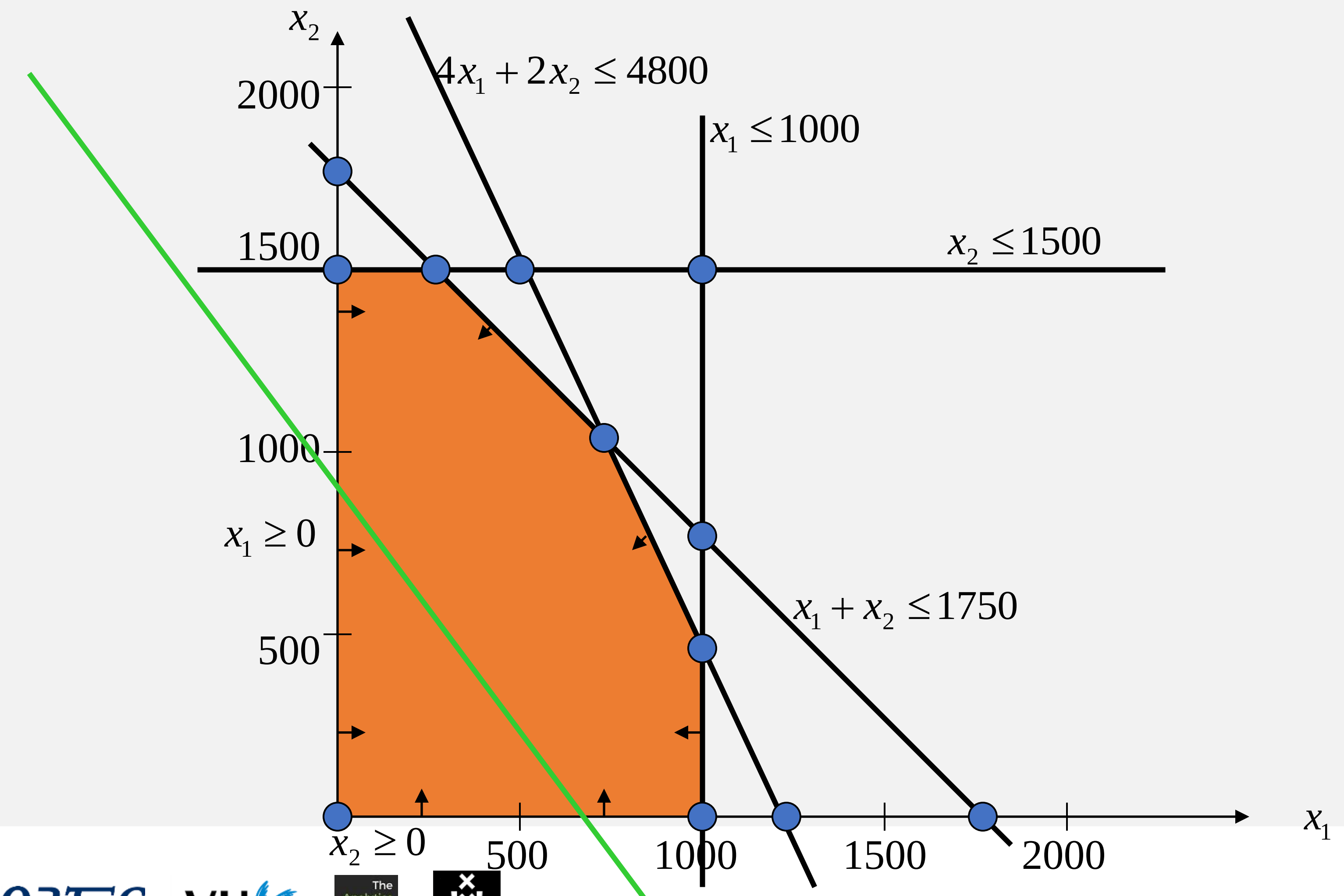
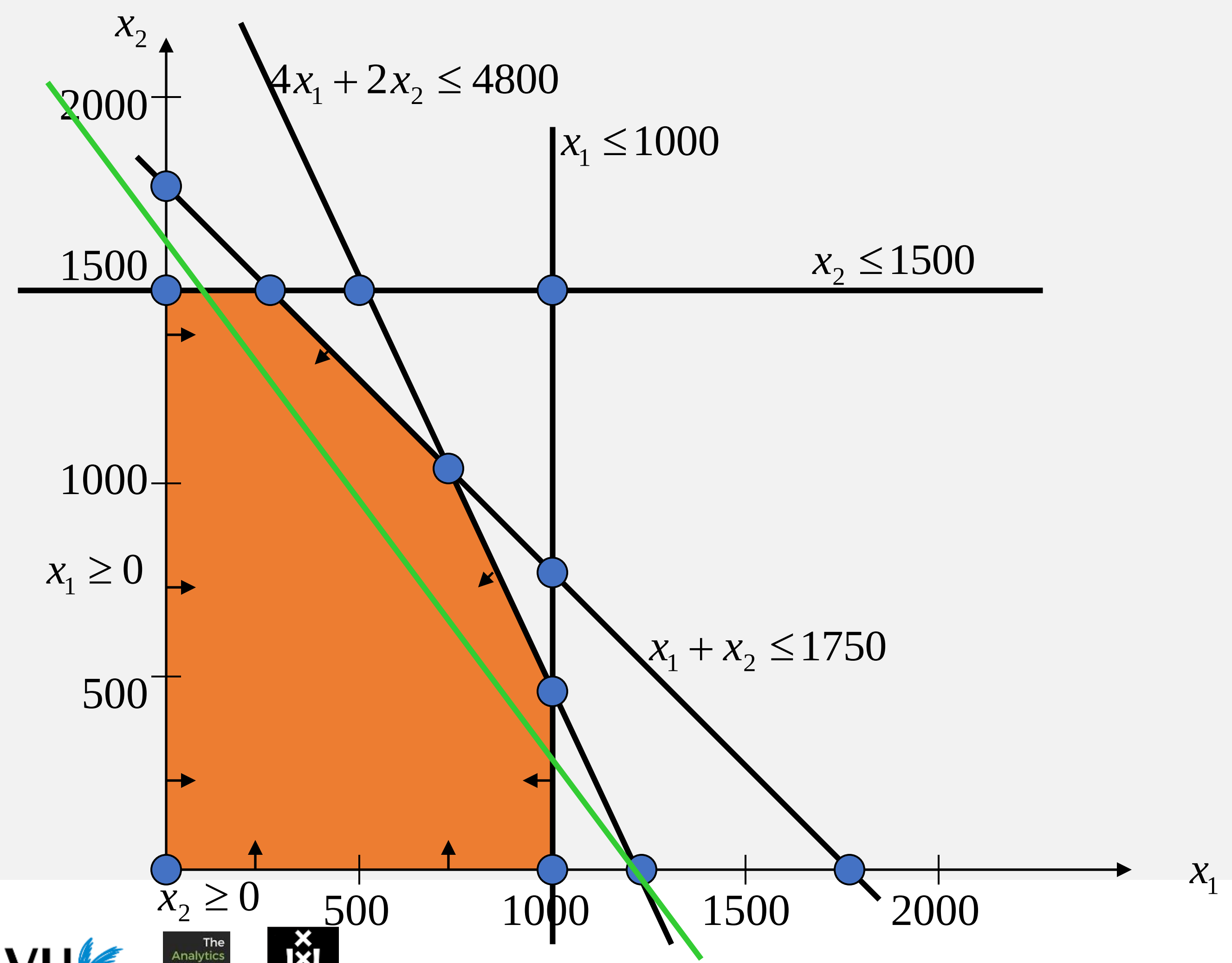
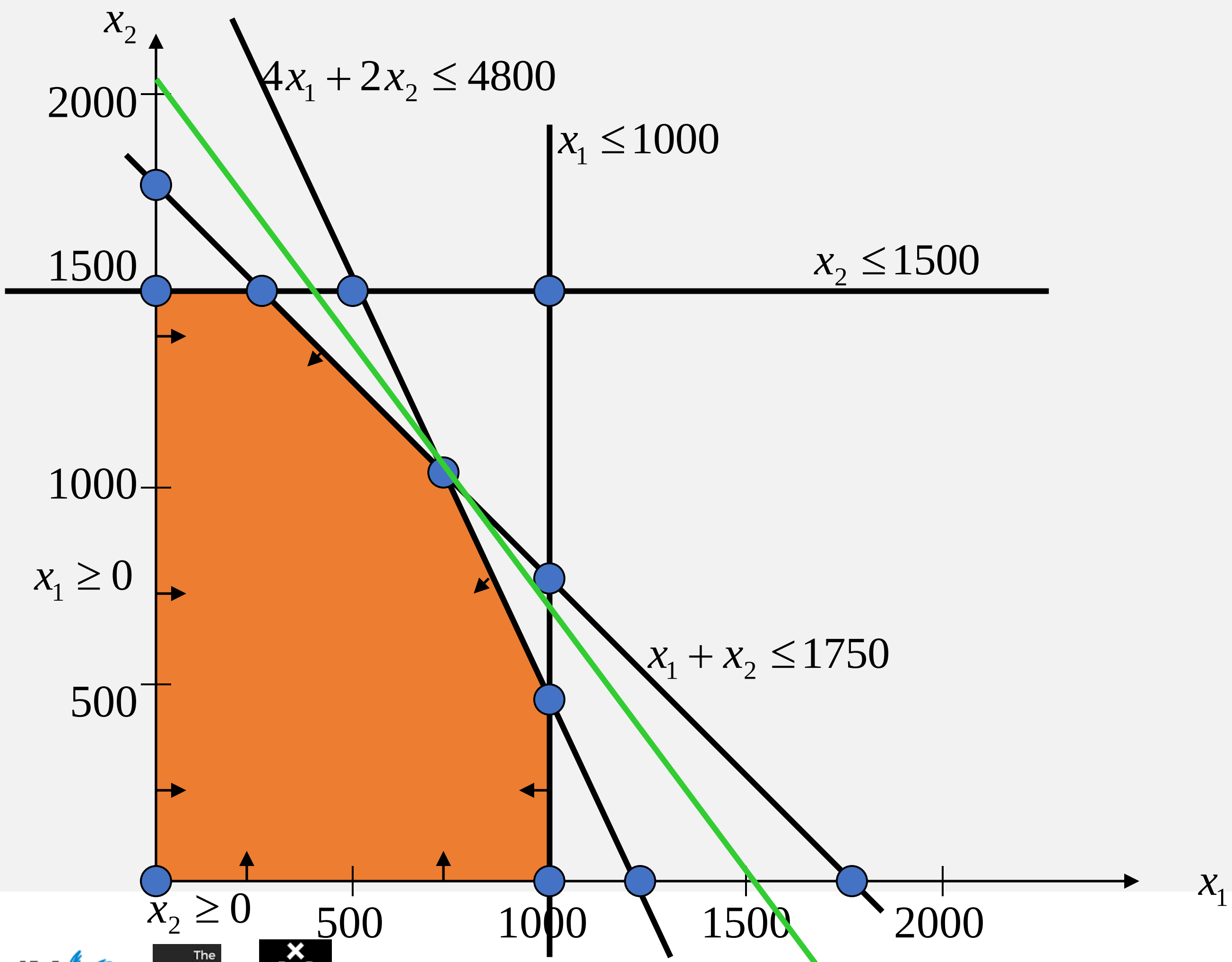


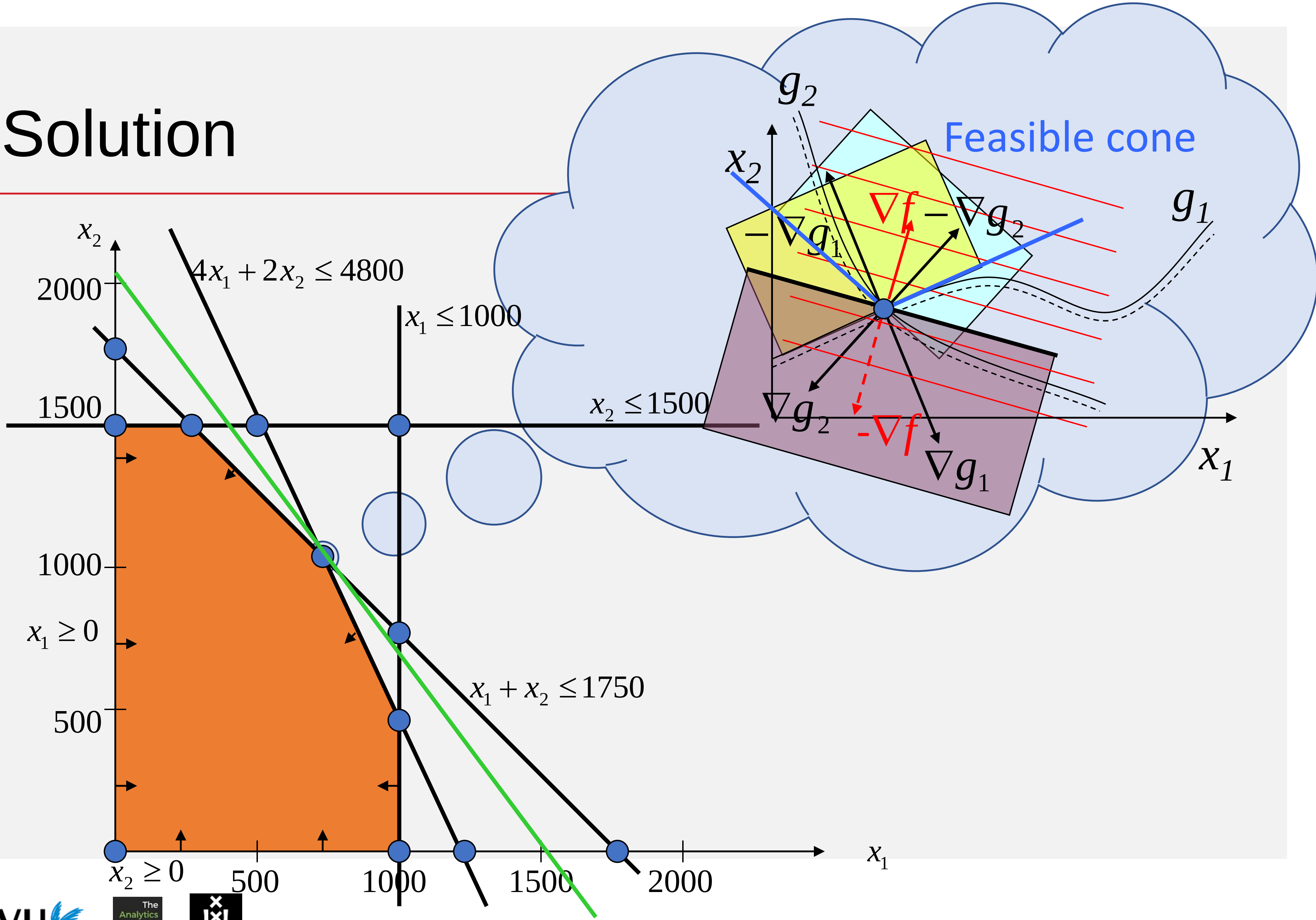
Graphical Solution



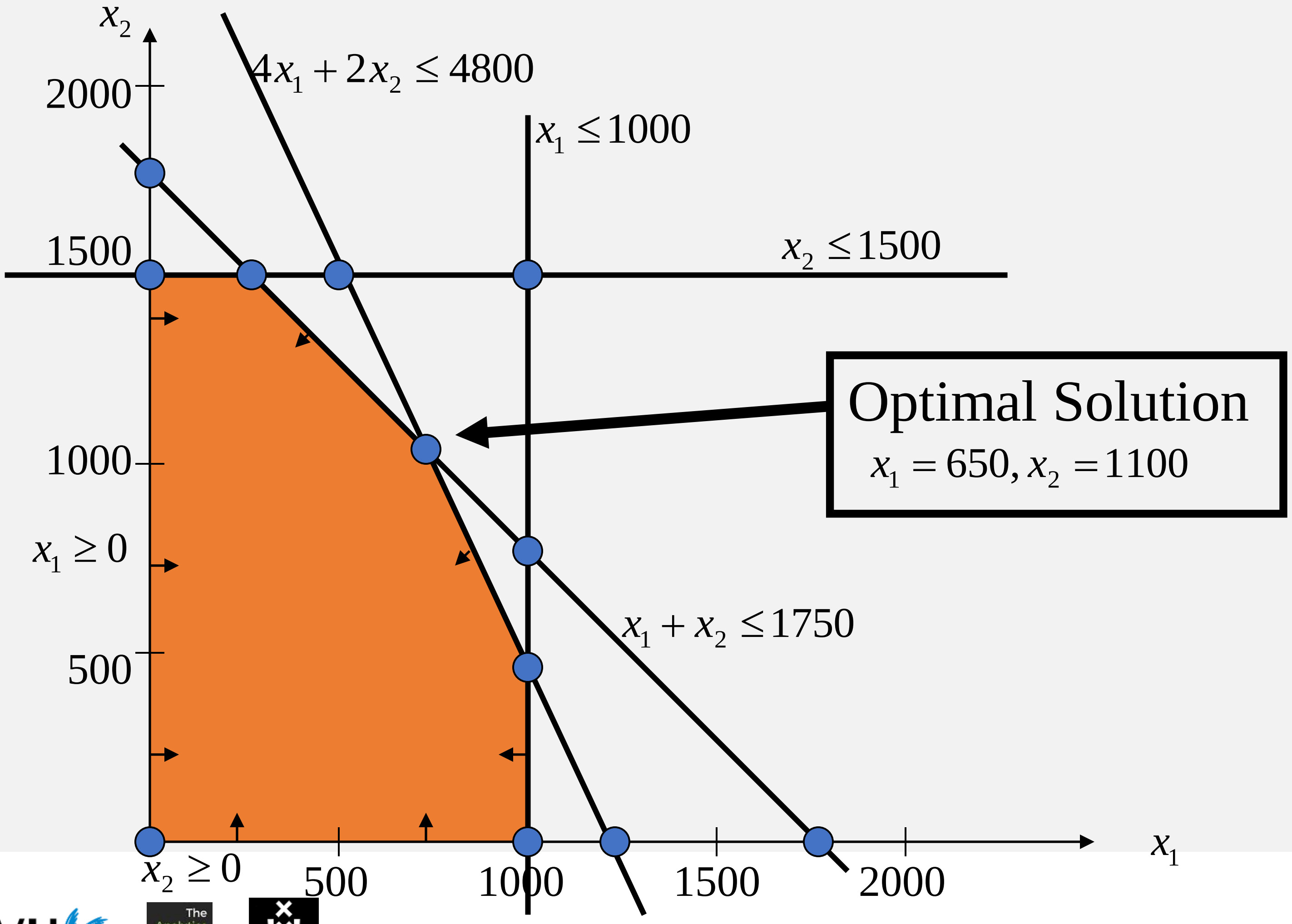
Graphical Solution



Graphical Solution



Graphical Solution



Optimal decision, prescription

- Achieve the maximum attainable profit of € 17.700
- producing 650 football trophies
- and producing 1.100 golf trophies
- while using all but 350 brass footballs,
- using all but 400 golf balls,
- using all the plaques,
- and using all of the wood.

This the moment to open the Caroline notebook

When the puzzle is feasibility

Optimization in disguise… and actually a tiny network flow model!

Dick’s table distribution

Caroline takes her associates and their families out to a dinner together at Dick’s restaurant to celebrate the successes brought by Mathematical Optimization. Dick ensures the required social distancing!

Family *f* has *m(f)* members.

At the restaurant, there are multiple tables, where table *t* has capacity *c(t).*

To increase their (**safe!**) social interaction, they would like to sit at tables so that <u>no more than *k* members of the same family are seated at the same table</u>.

Dick’s task is to distribute the families over the tables.

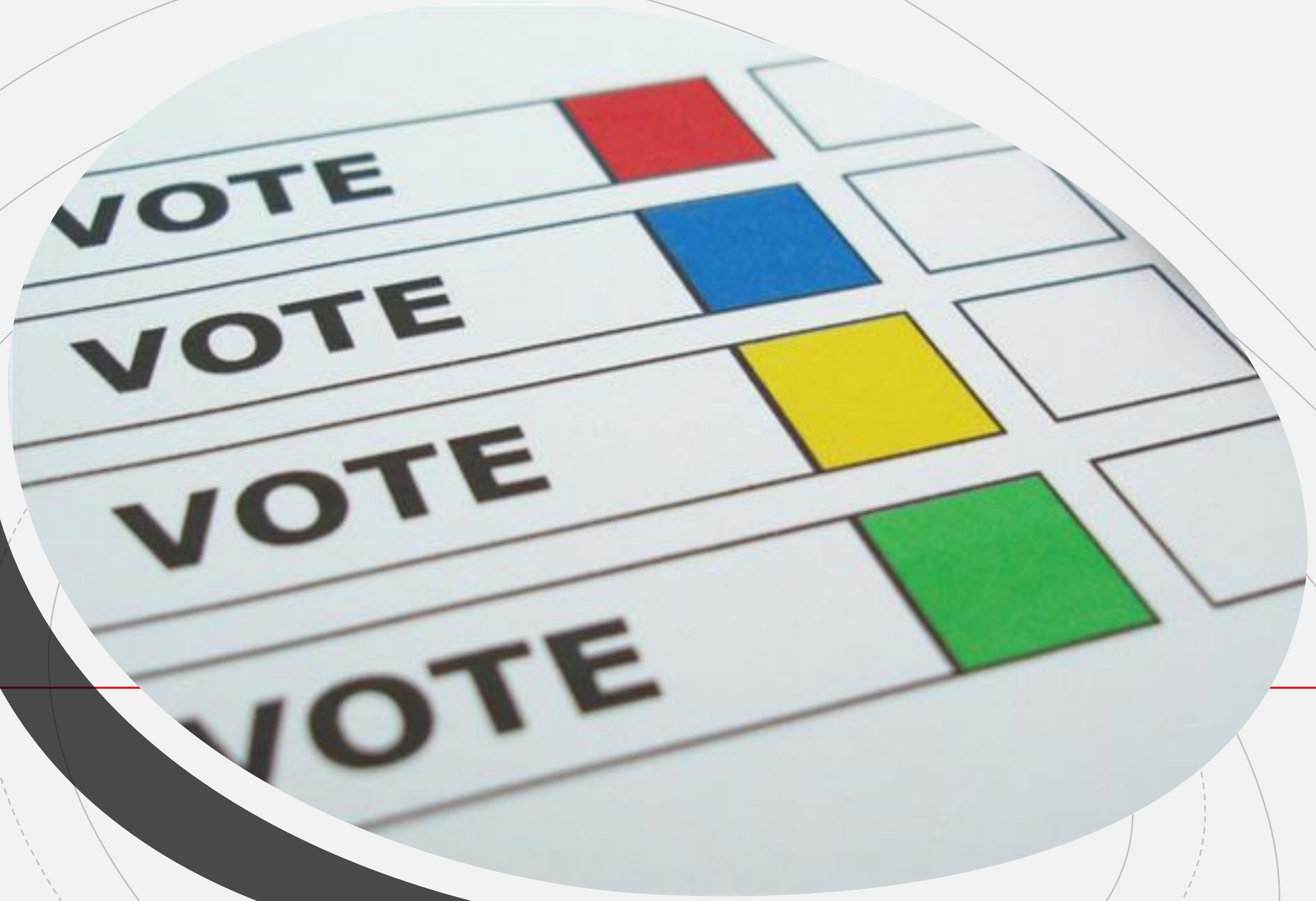
Model this problem and find a seating arrangement that satisfies this requirement, when the data is as follows:

m(f) = [6,8,2,9,13,1] , *c(t)* = [8,8,10,4,9], *k* = 3

→ Extra: also try to solve the model with *k* = 2. What should happen?

→ Extra: what you have more tables than needed? Can you minimize the number of tables used?

This the moment to open the Dick notebook



A second poll

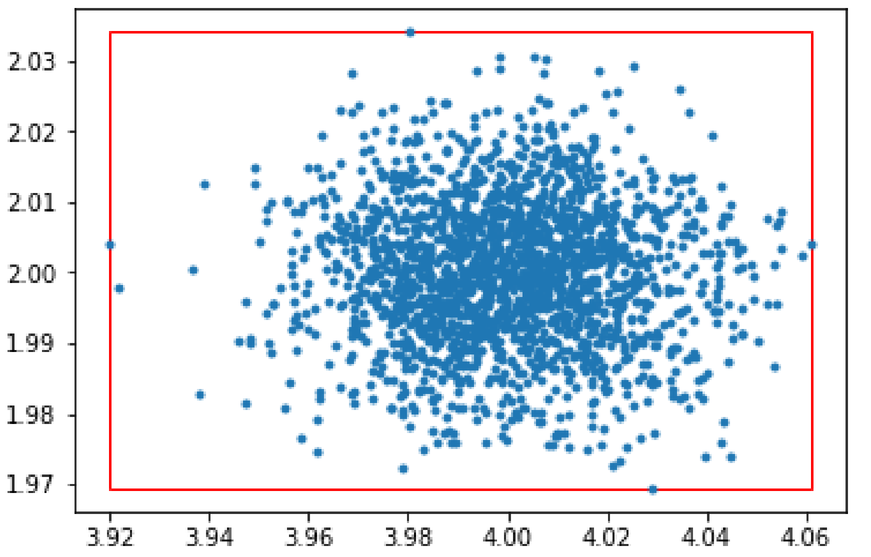
Do you always trust your data?

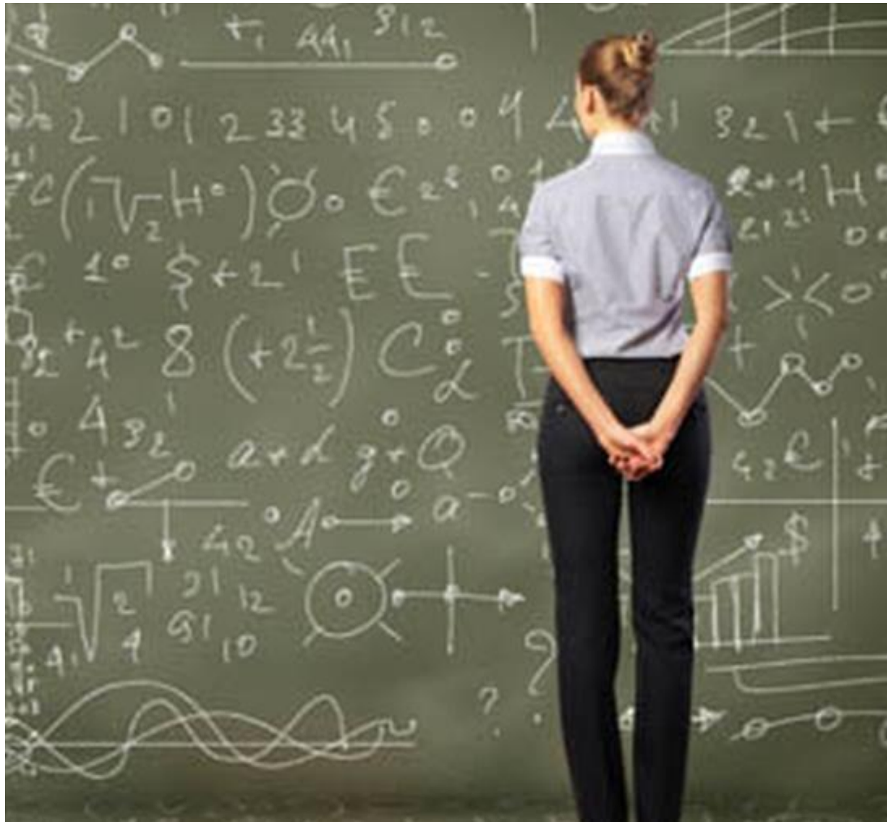
Suggestion

Consider watching Dick Den Hertog on "What every OR practitioner should know about Robust Optimization“ from the previous EURO congress in Dublin.

Els the data scientist

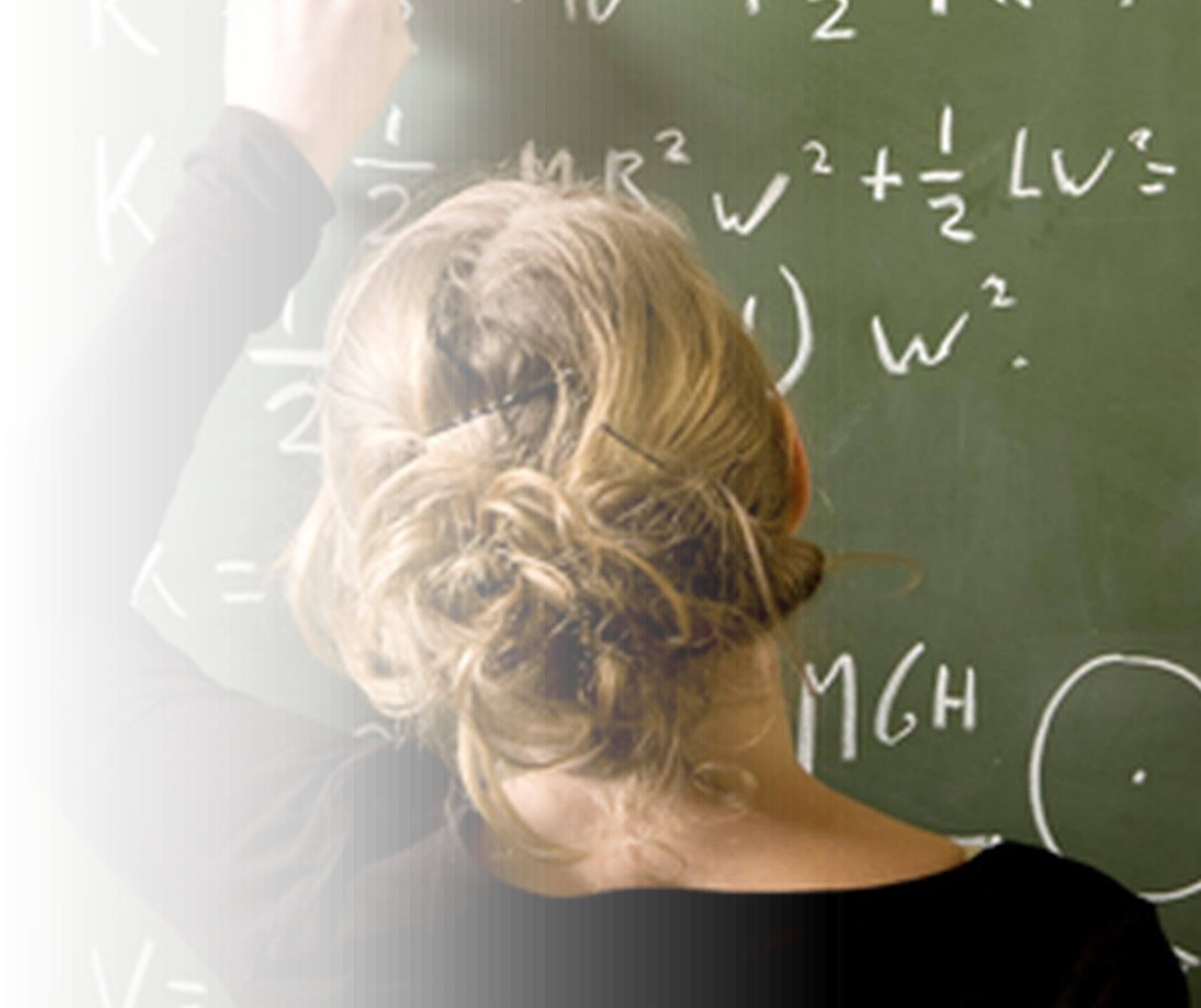
- Caroline hires Els to find out what the wood consumption really is.
- Els studies the past data and advises on an *uncertainty set*.





Fiona the expert optimizer

- Now that the uncertainty is clear, Caroline wants her production to be robust and she consults Fiona, the expert optimizer.



This the moment to open the Els and Fiona notebook

Models with different strengths

Same problem, different models

Caroline asks Gina to determine the best locations to install distribution centers

Gina’s expansion analysis

This the moment to open the Gina notebook

Become handy with data

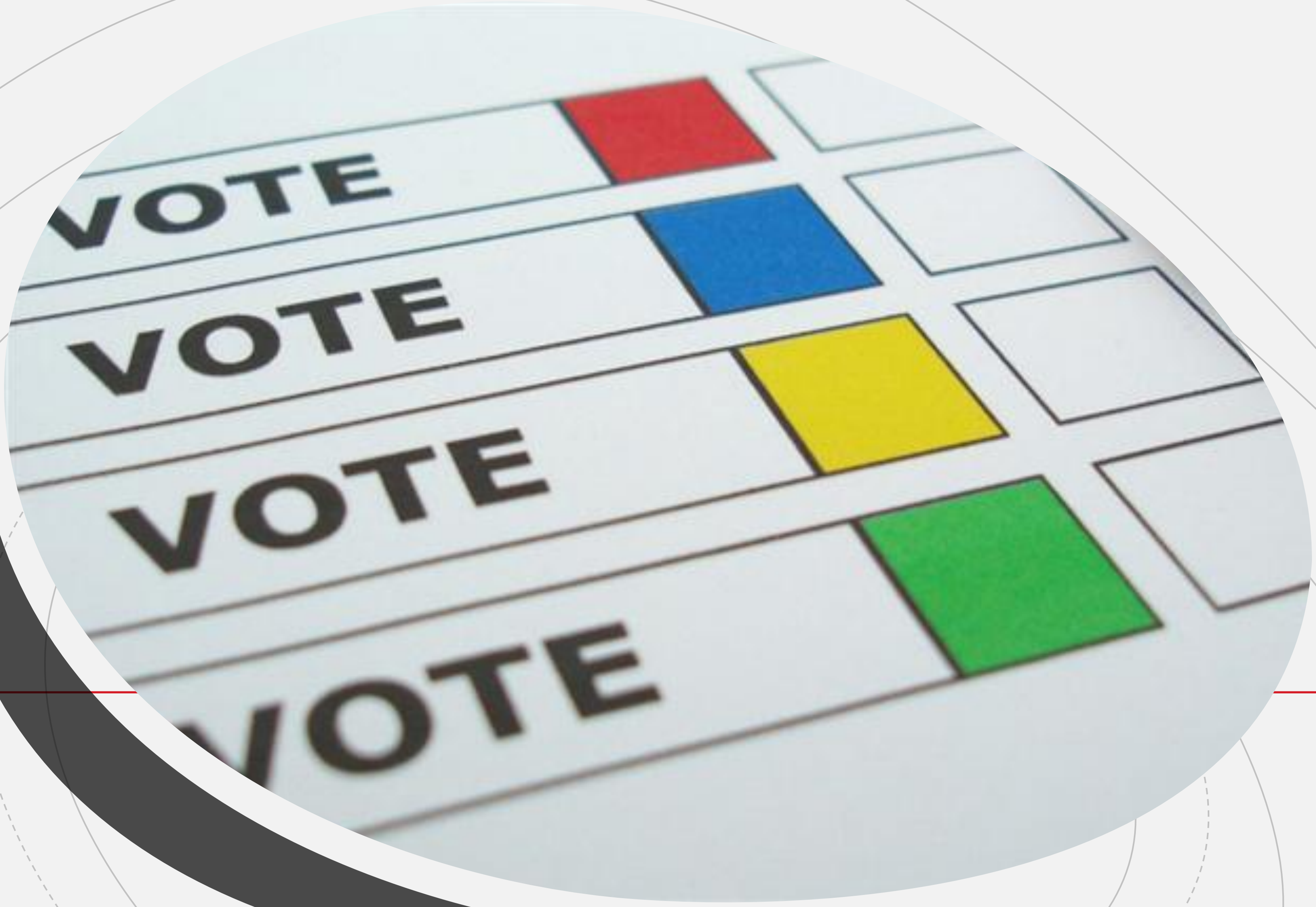
All data scientists love pandas… you should also!

Hilda’s material planning

- Caroline receives a forecast of demands for her products.
- She hires Hilda to optimize acquisition and inventory of the materials.
- There are three suppliers with different pricing schemes.
- All is summarized in the accompanying notebook!

Trophy	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Football	88	125	260	217	238	286	248	238	265	293	259	244
Golf	47	62	81	65	95	118	86	89	82	82	84	66

This the moment to open the Hilda notebook



A third poll

Are you missing routing and transportation too much?

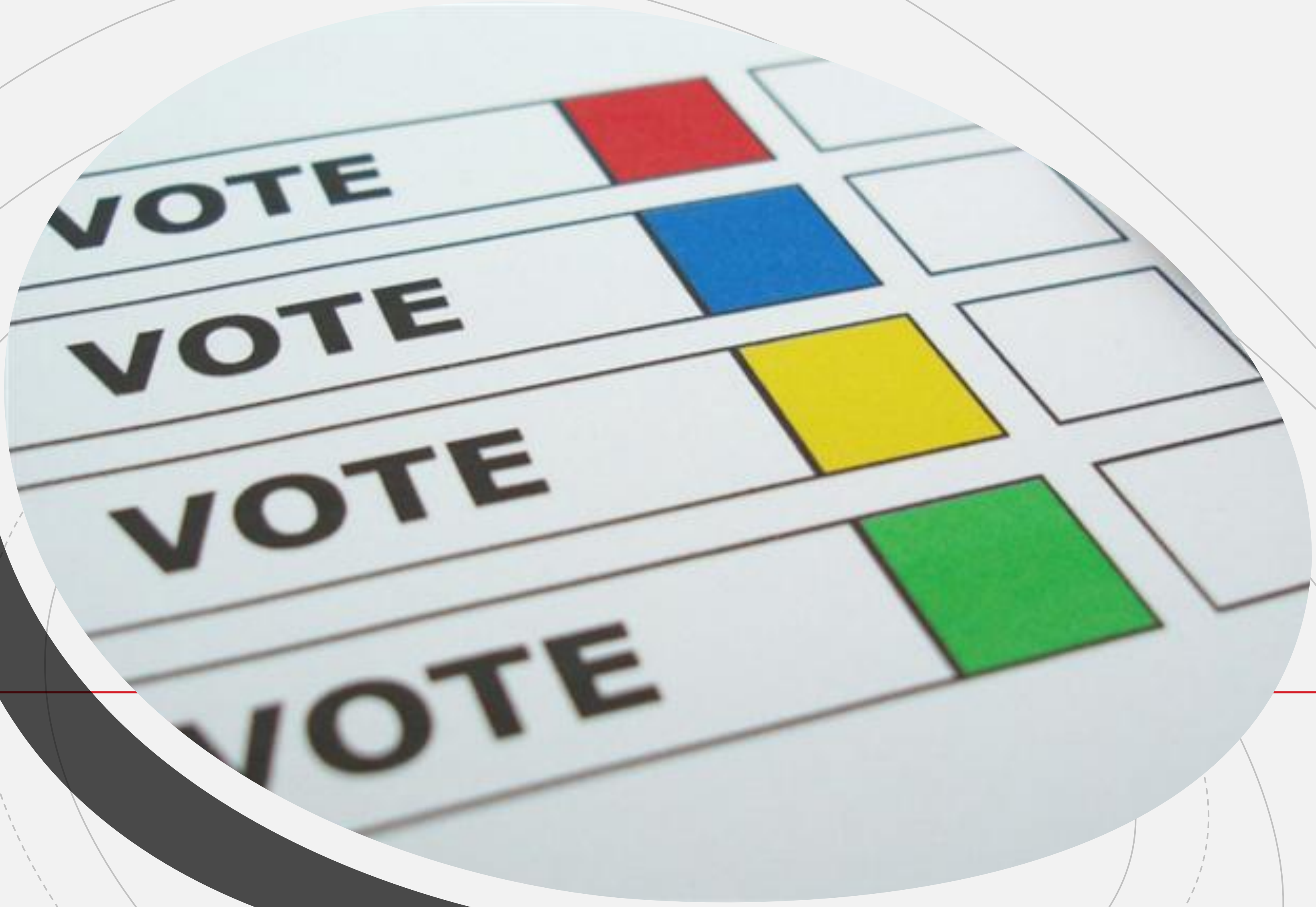
Suggestion

Consider watching [William Cook on "The Traveling Salesman Problem: postcards from the edge of impossibility"] also from the previous EURO congress.

Meet Igor, the Travelling Salesman

- Caroline hires Igor to acquire new customers.
- He needs to visit all potential costumers.
- His route should depart in the morning from his home and return there in the evening.
- To maximize the time at the costumers, Igor wants to spend the fewest possible time on the road.

This the moment to open the Igor notebook



A last poll

Did you like this?

May you wish to contact me:

joaquim.gromicho@ortec.com

Thank you very much!

Q&A

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