Method for Solving Fermatean Fuzzy Transportation Problem Using a Grade Activity

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Abstract

In this, we present an approach to the surplus and requirement and transit cost in a transportation problem which is not in a deterministic environment. Fermatean fuzzy set (FFS) is a more efficient, flexible, and generalized model to deal with uncertainty, as compared to intuitionistic and Pythagorean fuzzy models. This research present the decision maker has the option to using exact paramedic qualities. Now any case, actually, the decision maker will most likely be unable to set exact parametric qualities because of wild monetary climate. They have also introduced the concept of grade and veracity activity. In this paper, we explanation the grade activity and veracity activity. We solve transportation problems by considering the parameters by applying the transportation problem method of three generalised FFS. The aim of the Fermatean fuzzy numbers is to ease decision-making of unlimited values. For the first time, Fermatean fuzzy domain are used to solve a transportation problem. Some important and definition and properties of the proposed FFS are discussed, and to achieve the optimal alternative, the proposed Fermatean fuzzy transportation problems technique is carried out in a real life-application of the situations. An algorithm of the proposed technique is also developed. Then, we defined the transportation problem in FFS surroundings by accepting the system's proponent set and using the perfectly perceived POM QM excel solver. Further we developed which perform a crucial role in decision-making (DM). Then, show the exactness and validity of the proposed model and with the existing models is presented. Finally, the significance of the paper and scope of future study are mentioned.

Keywords

Pythagorean fuzzy numbers, Fermatean fuzzy numbers, Grade activity, Veracity activity, POM QM EXCEL SOLVER

1. Introduction

The main process of this work discuss to introduce fuzzy fermatean Transportation Problem, this is the extension part of Pythagorean fuzzy and after discussed with activity rate its grade value, veracity activity and its rate. Then we discussed relation between grades of fuzzy fernmatean function. Here we have converted into the fuzzy fermatean transportation problem. Then the problem defined mathematical formulation of linear transportation model, this model can be divided into five parts. Here proposed our method to extend the optimal solution of the fuzzy fermatean transportation problem and finally we represent although five model results and comparative study between each model.

The transportation problems to possess a birth of goods or shift of goods from source or store to a different area. The vision of the Fermatean fuzzy set was presented by Zadeh[2] as a tool able to manage mathematically the uncertainty. Zimmerman[16] was the first one to use the fuzzy notions in mathematical programming suggesting a fuzzy technique for linear programming with multi-objective functions. Latest, Zimmerman[15] was concerned with the applications of fuzzy collection theory to mathematical programming in a wider sense. More recently, Zadeh[3] readdress the fuzzy notions discussing the advantages of using fuzzy logic in solving real-life problems. Several articles that handle full fuzzy programming problems can be found in recent publications (see for instance [8]).

The Fuzzy Set theory, introduced by Zadech (1965), has been applied successfully in various fields[1]. The use of fuzzy set theory became very rapid in the field of optimization after the pioneering work done by Bellman and Zadeh (1970) [2]. With a addition of advantages, the theory of intuitionistic fuzzy sets was originated by Atanassov in 1986. It is characterised by the membership degree and non-membership degree, and fulfils a constraint that intuitionistic fuzzy sets has appeared as one of the valuable means for characterising uncertainty and vagueness in real-life problems [3]. Further, Yager originated the concept of the Pythagorean fuzzy set and extension of intuitionistic fuzzy sets expressed by the Membership and Non-Membership is restricted to one [4].

As a result, the Pythagorean fuzzy set doctrine is more desirable than the FSs doctrine (Zadeh 1965), as it has successfully demonstrated its strapping preparedness to deal with estimated and unpredictable instruction. As an extension of Fuzzy sets, the theory of Fermatean fuzzy sets (Senapathi and Yager 2019a) has been proven as one of the powerful platforms to deal with imprecise and unpredictable information [5]. Thus, the Fermatean fuzzy sets, Intuitionistic fuzzy sets, and Pythagorean fuzzy sets based on its particular more desirable, the paper focus of attention on the status. The theory of fuzzy sets has been insolently revised by researchers with enormous care for concerns with unpredictable and estimated instruction [7]. For example, Let A be the fuzzy set giving (0.9, 0.1, 0.3) where the degree of the membership is 0.01 and the degree of the non – membership is 0.22 and 0.77 other non-membership. The sum of the values of the degree of membership and non – membership is 1.3 > 1. This shows that *A* is not an Intuitionistic fuzzy set. Consider the sum of the

squares of the values of the degree of membership an non-membership is $(0.9)^2 + (0.1)^2 + (0.3)^2 > 1$ exceeding one which shows that *A* is not an Pythagorean fuzzy set. Consider the sum of the cubes of the values of the degree of membership and non–membership is $(0.9)^3 + (0.1)^3 + (0.3)^3 < 1$. Hence *A* is known as Fermatean Fuzzy set. Fermatean Fuzzy Sets is most effective than Pythagorean fuzzy sets and Intuitionistic Fuzzy Sets. As an increase of Intuitionistic fuzzy sets, Yager (2014) intimated the assumption of PFSs. The Pythagorean fuzzy sets are more capable tools than IFSs for handling the unpredictable and estimated information that arises in prudent problems. Newly, plentiful researchers have explored the distinct conception by seeing the theoretical and practical attitude of Pythagorean fuzzy sets [20,21]. The vacation of the document has been comprehensive as pursue.

The presented essential definition correlated to the Pythagorean fuzzy set, Fermatean Fuzzy set, Grade activity, Veracity activity in section 2. An improved the mathematical formulation for Fermatean Fuzzy set transportation problem in section 3. The solution expected for solving Fermatean Fuzzy Transportation problem and our proposal algorithm and using modified index numbers in section 4. And descriptive example, has been taken to showing the grade activity using modified index number of surplus and requirement and transit cost. The Fermatean fuzzy transportation problem development and analysis by an arithmetical descriptive with a numerical example in section 5.

2. Review of literature

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Baidya, A., Bera, U.K., Maiti, M.[31] presented a restricted multi-objective solid transportation problem with budget constraint involving stochastic variable and nterval type-2 fuzzy number. Ghosh, S., Küfer, K.-H., Roy, S.K., Weber, G.-W.[32] investigated carbon mechanism on sustainable multi-objective solid transportation problem for waste management in pythagorean hesitant fuzzy

environment. Bisht, M., Dangwal, R.[33] proposed a solving interval-valued transportation problem using a new ranking function for octagonal fuzzy numbers. Kacher, Y., Singh, P.[34] applied Fuzzy harmonic mean technique for solving fully fuzzy multi-objective transportation problem. Halder (Jana), S., Jana, B.[35] discussed the Application of fuzzy logic based GA and PSO to solve 4D multi-item transportation problem for substitute and complementary items.

Vidhya, V., Ganesan, K.[36] formulated a new ranking approach for solving fuzzy transportation problem with pentagonal fuzzy number. Sanjana, R., Ramesh, G.[37] presented a novel approach to interval-valued variables using new interval arithmetic to solve an intuitionistic fuzzy transportation problem. Dhanasekar, S., Rani, J.J., Annamalai, M.[38] defined a transportation problem for interval-valued trapezoidal intuitionistic fuzzy numbers. Saini, R., Joshi, V.D., Singh, J.[39] proposed On solving a MFL paradox in linear plus linear fractional multi- objective transportation problem using fuzzy approach. Prabhavati, B.S.S., Ravindranath, V.[40] developed a simple and efficient method to solve fully interval and fuzzy transportation problems. Bisht, M., Dangwal, R.[41] elongated it fuzzy approach to solve interval-valued transportation problem and comparison of the effectiveness of differentfuzzy numbers.

Akram, M., Shah, S.M.U., Ali Al-Shamiri, M.M., Edalatpanah, S.A. [42] defined fractional transportation problem under interval-valued fermatean fuzzy sets. Mehmood, M.A., Bashir, S. [43] proposed extended transportation models based on picture fuzzy sets. Ghosh, S., Roy, S.K., Fügenschuh, A. [44] derived the multi-objective solid transportation problem with preservation technology using pythagorean fuzzy sets. Radhika, K., Arun Prakash, K.[45] observed a multiobjective optimization for multi-type transportation problem in intuitionistic fuzzy environment. Anukokila, P., Radhakrishnan, B.[46] was studied extensively from the goal programming approach in profit maximization of solid fractional transportation problem with interval type-2 fuzzy numbers. Das, K.N., Das, R., Acharjya, D.P.[47] investigated least-looping stepping-stone-based ASM approach for transportation and triangular intuitionistic fuzzy transportation problem. El Sayed, M.A., El-Shorbagy, M.A., Farahat, F.A., Fareed, A.F., Elsisy, M.A.[48] formulated stability of parametric intuitionistic fuzzy multi-objective fractional transportation problem. Vidhya, V., Uma Maheswari, P., Ganesan, K.[49] have presented an alternate method for finding more for less solution to fuzzy transportation problem with mixed constraints. Mahla, D., Agarwal, S., Mathur, T. [50] developed the novel fuzzy non-radial data envelopment analysis: an application in transportation. Arora, R., Gupta, K.[51] used the fuzzy programming for multi-choice bi-level transportation problem. Mahla, D., Agarwal, S.[52] he basic of A fuzzy DEA approach on the extended transportation problem. Gurukumaresan, D., Duraisamy, C., Srinivasan, R.[53] investigated optimal solution of fuzzy transportation problem using octagonal fuzzy numbers. Ghosh, S., Roy, S.K. [54] employing Fuzzyrough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station. Sam'an, M., Farikhin. [55] originated A new fuzzy transportation algorithm for finding fuzzy optimal solution.

Muthuperumal, S., Titus, P., Venkatachalapathy, M. [56] discussed and further attained of a An algorithmic approach to solve unbalanced triangular fuzzy transportation problems. Pathade, P.A., Hamoud, A.A., Ghadle, K.P.[57] currently a series of methods a systematic approach for solving mixed constraint fuzzy balanced and unbalanced transportation problem. Hedid, M., Zitouni, R.[58] improved to solving the four index fully fuzzy transportation problem. Veeramani, C., Robinson, M.J., Vasanthi, S.[59] in this section we propound the related work value and ambiguitybased approach for solving intuitionistic fuzzy transportation problem with total quantity discounts and incremental quantity discounts.

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Kumar, R., Edalatpanah, S.A., Jha, S., Singh, R.[65] put forward A Pythagorean fuzzy approach to the transportation problem. Maity, S., Roy, S.K.[66] in addition defined a new approach for solving type-2-fuzzy transportation problem. Mahmoodirad, A., Allahviranloo, T., Niroomand, S.[67] is a famous model a new effective solution method for fully intuitionistic fuzzy transportation problem. Baykasoğlu, A., Subulan, K.[68] used the conversion of a direct solution approach based on constrained fuzzy arithmetic and metaheuristic for fuzzy transportation problems. Anukokila, P., Anju, A., Radhakrishnan, B.[69] gave an optimality of intuitionistic fuzzy fractional transportation problem of type-2. Anukokila, P., Radhakrishnan, B. [70] solved goal programming approach to fully fuzzy fractional transportation problem efficiency of the multi-objective proposed approach. Yankova, T., Ilieva, G.[71] was credited for proposing solving a fuzzy transportation problem based on exponential membership functions. Malini, P.[72] generalized A new ranking technique on heptagonal fuzzy numbers to solve fuzzy transportation problem.

Anukokila, P., Radhakrishnan, B., Anju, A.[73] was the first goal programming approach for solving multi-objective fractional transportation problem with fuzzy parameters. Ebrahimnejad, A., Verdegay, J.L.[74] initiated a new approach for solving fully intuitionistic fuzzy transportation problems. Bharati, S.K., Singh, S.R.[75] then extended the transportation problem under interval-valued intuitionistic fuzzy environment. Das, A., Bera, U.K., Maiti, M.[76] used the conversion of defuzzification and application of trapezoidal type-2 fuzzy variables to green solid transportation problem. Shojaie, A.A., Raoofpanah, H.[77] extended solving a two-objective green transportation problem by using meta-heuristic methods under uncertain fuzzy approach. Kar, M.B., Kundu, P.,

Kar, S., Pal, T.[78] introduced a multi-objective multi-item solid transportation problem with vehicle cost, volume and weight capacity under fuzzy environment.

2. Preliminaries

Some elemental explanation about Pythagorean fuzzy sets and Fermatean fuzzy sets are given in this section.

2.1. Pythagorean fuzzy sets:[13,14]

The Pythagorean fuzzy sets defined on a non-empty set X as objects having the form

 $P = \{ \langle x, \alpha_{P(x)}, \beta_{P(x)}, \gamma_{P(x)} \rangle : x \in X \},\$

where $\alpha_{P(x)}: X: \to [0,1]$ and $\beta_{P(x)}: X: \to [0,1]$ and $\gamma_{P(x)}: X: \to [0,1]$, with the condition

 $0 \le (\alpha_{P(x)})^2 + (\beta_{P(x)})^2 + (\delta_{P(x)})^2 \le 1, \forall x \in X.$ The values $\alpha_{P(x)}$ and $\beta_{P(x)}$ and $\gamma_{P(x)}$ denote,

respectively, the degree of membership and the non-membership of the element $x \in X$ in the PFS *P* For any Pythagorean fuzzy set *P* and

$$x \in X, \pi_{P(x)} = \sqrt{1 - (\alpha_{P(x)}) - (\beta_{P(x)}) - (\delta_{P(x)}) - (\alpha_{P(x)})' - (\beta_{P(x)})' - (\delta_{P(x)})'}$$
 is called the

degree of hesitancy or the degree of indeterminacy of x to P.

2.1.1. Definition[13,14]

Let $P = \langle \alpha_P, \beta_P, \gamma_P \rangle$ be a Pythagorean fuzzy set, then the Grade activity of *P* stand for by $R_P(P)$ and is detailed during the time that $R_P(P) = \frac{1}{2}(1 + \alpha_P^2 - \beta_P^2 - \gamma_P^2)$

2.1.2. Definition [13,14]

Let $P = \langle \alpha_P, \beta_P, \gamma_P \rangle$ be a Pythagorean fuzzy set, then the veracity activity of *P* is stand for by $Q_P(P)$ and is detailed during the time that $Q_P(P) = \alpha_P^2 + \beta_P^2 + \gamma_P^2$

2.2. Fermatean fuzzy sets[8, 9]

Let X be a non-empty universe of discussion. A FFs in F in X is an object

with the following shape: F={ $(x, \alpha_P(x), \beta_P(x), \gamma_P(x)): x \in X$ }

Where $\alpha_P(x): X \to [0,1], \ \beta_P(x): X \to [0,1], \ \gamma_P(x): X \to [0,1]$ which includes the circumstance $0 \le [(\alpha_{P(x)})^3 + (\beta_{P(x)})^3 + (\delta_{P(x)})^3] \le 1 \ \forall x \in X.$

The functions $\alpha_P(x)$, $\beta_P(x)$ and $\gamma_P(x)$ denote respectively the degree of membership and the degree of non-membership of the element *x* in the set F. For any Fermatean fuzzy set F and $x \in X$,

$$\pi_P(x) = \sqrt{1 - (\alpha_{P(x)}) - (\beta_{P(x)}) - (\delta_{P(x)}) - (\alpha_{P(x)})' - (\beta_{P(x)})' - (\delta_{P(x)})'}$$
 is known as the

degree of indeterminancy of x to F.

2.2.1. Definition[15,16]

Let $P = \langle \alpha_{P_1}, \beta_{P_1}, \gamma_{P_1} \rangle$, $P_1 = \langle \alpha_{P_1}, \beta_{P_1}, \gamma_{P_1} \rangle$, $P_2 = \langle \alpha_{P_2}, \beta_{P_2}, \gamma_{P_2} \rangle$, be three Fermatean fuzzy sets and $\lambda > 0$, then their operations are interpreted in this ways.

i)
$$P_1 + P_2 = \langle \sqrt[3]{(\alpha_{P_1})^3 + (\alpha_{P_2})^3 - (\alpha_{P_1})^3 (\alpha_{P_2})^3, \beta_{P_1} \beta_{P_{2,'}}, \gamma_{P_1} \gamma_{P_2} \rangle}$$

ii)
$$P_1 \times P_2 = \langle \alpha_{p_1} \alpha_{p_2}, \sqrt[3]{\beta_{P_1}^3 + \beta_{P_2}^3 - \beta_{P_1}^3} \beta_{P_2}^3, \gamma_{P_1} \gamma_{P_2} \rangle$$

iii) $\lambda P = \langle \sqrt[3]{1 - (1 - \alpha_P^3)^{\lambda}}, (\beta_P)^{\lambda} (\gamma_P)^{\lambda} \rangle$
iv) $P^{\lambda} = \langle \alpha_P^{\lambda}, \sqrt[3]{1 - (1 - \beta_P^3)^{\lambda}}, \gamma_P^{\lambda} \rangle$

2.2.2. Definition[10]

Let $P = \langle \alpha_{P_1} \beta_{P_1} \gamma_{P_1} \rangle$, $P_1 = \langle \alpha_{P_1} \beta_{P_1} \gamma_{P_1} \rangle$, $P_2 = \langle \alpha_{P_2} \beta_{P_2} \gamma_{P_2} \rangle$ be three Fermatean fuzzy sets closed the nonempty everything X, then their operation are detailed as pursue:

- (i) $P_1 \cap P_2 = \langle \min\{\alpha_{P_1}, \alpha_{P_2}\}, \max\{\beta_{P_1}, \beta_{P_2}\}, \min\{\gamma_{P_1}, \gamma_{P_2}\} \rangle$
- (ii) $P_1 \cup P_2 = \langle min\{\alpha_{P_1}, \alpha_{P_2}\}, max\{\beta_{P_1}, \beta_{P_2}\}, min\{\gamma_{P_1}, \gamma_{P_2}\} \rangle$
- (iii) $P_1^{\ c} = \langle \alpha_{P_1}, \beta_{P_1}, \gamma_{P_1} \rangle$

2.2.3. Definition [8, 9]

Let $P = \langle \alpha_P, \beta_P, \gamma_P \rangle$ be a Fermatean fuzzy set then the grade activity P can be defined as

$$\mathbf{P} = \alpha_P^3 - \beta_P^3 - \gamma_P^3$$

The grade rate between lies in the interval $[-1,1] R_P(P) \in [-1,1]$. Its positive when $R_P(P) \in [0,1]$ and negative when $R_P(P) \in [-1,0)$ In the approach of ranking of Fermatean fuzzy numbers (Intuitionistic fuzzy set or Pythagorean set) more than research have suggests grade activity whose grade rate are falsity between in the interruption 0 and 1.

(i)
$$G_{1P}(P) = \frac{1}{2} \left(1 + \alpha_P^3 - \beta_P^3 - \gamma_P^3 \right)$$

(ii)
$$G_{2P}(P) = \frac{1}{3} \left(1 + 2\alpha_P^3 - \beta_P^3 - \gamma_P^3\right)$$

(iii) $G_{3P}(P) = \frac{1}{2} \left(1 + \alpha_P^3 - \beta_P^3 - \gamma_P^3 \right) \left| \alpha_P - \beta_P - \gamma_P \right|$

2.3. Property:

For any FFS $G_P = \langle \alpha_P, \beta_P, \gamma_P \rangle$ the suggested grade function $(G_P) \in [-1, 1]$

Proof:

We know that for FFS, $\alpha_F^3 + \beta_F^3 + \gamma_F^3 \le 1$, Thus $\alpha_F^3 - \beta_F^3 - \gamma_F^3 \ge -1$ Hence $-1 \le [\alpha_F^3 - \beta_F^3 - \gamma_F^3] \le 1$ namely grade (G_P) \in [-1, 1]. In particular, if G_P = (0,1), then grade (G_P)= -1, if G_P = (1,0), then grade (G_P) = 1

2.3.1 Grade Activity

Fuzzy sets are defined with corresponding to a non empty basic set X of elements of functions. The important information about that each quantity of $x \in X$ is defined a membership grade activity Grade (*F*) to set A, taking values in [0,1] with Grade (*F*) =0 corresponding to non-

membership. Figure 1 shows that grade activity ranges from 0 < Grade(F) < 1 to membership grade, and Grade not (F) = 1 to complete membership.

Grade of membership (F) = $\begin{cases} 0 & x < -0.6\\ 0.5x - 0.1 & 0.1 < x \le 0.7\\ 1 & x > 0.7 \end{cases}$

Grade not membership (F) =
$$\begin{cases} 1 & x \le 0.1 \\ -0.5x + 0.1 & 0.1 < x \le 0.7 \\ 0 & x > 0.7 \end{cases}$$

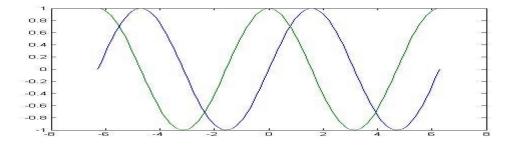


Figure 1. Grade membership activity

2.3.2. Veracity activity

Let L= $\langle \alpha_i, \gamma_i, \beta_i, \delta_i \rangle$ a be any Fermatean Fuzzy Set, then Legitimacy function of *L* characterized by $L_{\mathbb{R}}$ and is described as

$$L_{\mathbb{R}} = \frac{2(RC + SD) - (R + S) + 2}{4} \quad i = 1, 2, 3, ..., l$$

Veracity activity is a methodology to calculate the truthfulness, capacity,requirement and authentically when data refers to the path of rectifying whether the value or the overall cost is truth or not. The representation for FFN as $F=(r_f,s_f)$, where r_f and $c_f \in [0,1]$ this known as truth and falsity of the commencement of F respectivily, there is relation to membership and non-membership given by $\mu_p = r_f cos\theta_f$ and $\vartheta_{p=} r_f sin\theta_f$, where veracity activity $\theta_f \rightarrow \arccos(\frac{\mu_p}{r_f})$ and $s_f=1-\frac{20p}{\pi}$, veracity of Maximum and Minimum $R_i = \frac{1}{2}(1 + \alpha_i^3 - \beta_i^3), S_i = \frac{1}{2}(1 + \gamma_i^3 - \delta_i^3)$, direction of the veracity (Maximum and Minimum)

 $C_i = 1 - \frac{2X}{\pi}$, $D_i = 1 - \frac{2Y}{\pi}$; Where $X = \sec\left(\frac{\beta}{R}\right)$, $Y = \sec\left(\frac{\delta}{R}\right)$. Here L is the legitimacy of veracity function and the stability veracity of membership degree is obtained by RSC and D, where R and S is the stability of maximum and minimum, C and D is the direction of maximum and minimum stability. The variables α and β are maximum and minimum membership degrees, and γ and δ is the maximum and minimum non-membership degrees. The other variables like X and Y are the direction of $\sec\left(\frac{\beta}{R}\right)$ and $\sec\left(\frac{\delta}{R}\right)$. **2.3.3. Definition[8.9]** Let $F = \langle \alpha_F, \beta_F, \gamma_F \rangle$ be an FFS then the accuracy function of F is defined as Ver (F) = $\alpha_F^3 + \beta_F^3 + \gamma_F^3$

2.3.4. Definition [8, 9]

Let F = $\langle \alpha_F, \beta_F, \gamma_F \rangle$ be any FFS, then Grade activity of F denoted by Grade (*F*₁) and Grade F is defined as Grade (*F*₁), Grade (*F*₂), Grade (*F*₃) follows

- 1. If Grade $(F_1) <$ Grade (F_2) , then $F_1 < F_2$;
- 2. If Grade $(F_1) >$ Grade (F_2) , then $F_1 > F_2$;
- 3. If Grade (F_1) = Grade (F_2) , then $F_1 = F_2$

Let $F = (\alpha_F, \beta_F, \gamma_F)$ be any FFS, then Veracity activity of F denoted by Grade (F_1) and F is defined as Veracity (F_1), Veracity (F_2), Veracity (F_3) follows

- 1. If $Veracity(F_1) < Veracity(F_2)$, then $F_1 < F_2$;
- 2. If $Veracity(F_1) > Veracity(F_2)$, then $F_1 > F_2$;
- 3. If Veracity(F_1) = Veracity(F_2), then $F_1 = F_2$

3. Mathematical Formulation

In this section, the Transportation problem to expand the implementation of the transport production from v element to w target then the overall transit cost is diminishing. Then the association has v item and w channels. A particular production is to be developed from the item to the channel. Further made-up that any item has a given amount of supply. Further made-up that all bin has a liable of surplus and all channel has a liable amount of planned. Present made-up that any item has a given amount of supply. Further made-up that all bin has a liable of surplus and all channel has a liable amount of planned. Present they accept further made-up that the transportation cost of a entity duration of production against i^{th} element to j^{th} target is K_{ij} and the supply at i^{th} element is p_i and the demand of at j^{th} target is q_j and Y_{ij} is the number of entity the transportation cost entity duration. Then we along with assumed that the transit cost of a entity duration about particulars i^{th} element to j^{th} target and y_{ij} is the number of entity information and production developed from i^{th} element to j^{th}

3.1. Model 1

$$\mathbf{Minimize} \ \mathbf{z} = \sum_{i=1}^{v} \sum_{j=1}^{w} \mathbf{K}_{ij} \mathbf{Y}_{ij} \tag{1}$$

Subject to

$$\begin{split} & \sum_{j=1}^{w} y_{ij} \leq p_{i,} \ \text{i=1, 2...,v} \\ & \sum_{i=1}^{v} y_{ij} \geq q_{j}, \ \text{j=1, 2...,w} \end{split}$$

In the present issue the decision maker has the option to using exact paramedic qualities. Now any case, actually, the decision maker will most likely be unable to set exact information qualities. Thus,

in this issue, every one of the boundaries related with TP are viewed as FFSs. Furthermore, the FFSs are of the structure $\langle \alpha, \beta, \gamma \rangle$ where $0 \le \alpha^3 + \beta^3 + \gamma^3 \le 1$ Presenting imperativeness in customary TP, FFTP is planned as pursue.

3.2. Model 2

Minimize
$$\langle \alpha_{z_0}, \beta_{z_0}, \gamma_{z_0} \rangle = \sum_i^v \sum_j^v \left\langle \alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}} \right\rangle \odot K_{ij}$$
 (2)

Subject to

$$\begin{split} \sum_{j=1}^{w} y_{ij} &\leq \langle \alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} \rangle, \, i=1, 2, \dots, v \\ \sum_{i=1}^{v} y_{ij} &\geq \langle \alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} \rangle, \, j=1, 2, \dots, w \\ \text{Where } & 0 &\leq \left(\alpha_{z_0}\right)^3 + \left(\beta_{z_0}\right)^3 + \left(\gamma_{z_0}\right)^3 \leq 1 \\ & 0 &\leq \left(\alpha_{z_0}\right)^3 + \left(\beta_{z_0}\right)^3 + \left(\gamma_{z_0}\right)^3 \leq 1, \, i=1, 2, \dots, v \\ & 0 &\leq \left(\alpha_{z_0}\right)^3 + \left(\beta_{z_0}\right)^3 + \left(\gamma_{z_0}\right)^3 \leq 1, \, j=1, 2, \dots, w \\ & x_{ij} \geq 0 \text{ and } 0 \leq \left(\alpha_{z_0}\right)^3 + \left(\beta_{z_0}\right)^3 + \left(\gamma_{z_0}\right)^3 \leq 1, \, i=1, 2, \dots, v; \, j=1, 2, \dots, w \end{split}$$

From the table $\langle \alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} \rangle$ is the overall Fermatean fuzzy opportunity of the information at *i*th element, to $\langle \alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} \rangle$ is the total Fermatean fuzzy transportation expenditure from *i*th element to j^{th} target, $\langle \alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}} \rangle$ is entity Fermatean fuzzy transportation expenditure against *i*th element to *j*th target.

The numercial version of a transportation hassle in Fermatean fuzzy surroundings referred to as Fermatean fuzzy transportation hassle. It is to be referred to that if $\sum_{i=1}^{m} \bigoplus \langle \alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} \rangle = \sum_{j=1}^{n} \bigoplus \langle \alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} \rangle$. Prescription Data for transit cost, surplus and requirement stated to be equivalent, in any other case its miles referred to as unequivalent FFSTP hassle. The design $\sum (p \bigoplus q)/N$ stand for as aggregate price and quantity index number in phrases of Fermatean extension impression [17, 18].

Table 1

	Ι	II	III	IV	Supply
Z1	<0.1,0.2, 0.7>	<0.1, 0.2, 0.9>	<0.1, 0.3, 0.9>	<0.1, 0.2, 0.6>	<0.1,0.2, 0.5>
Z2	<0.01,0.22, 0.77>	<0.1, 0.2, 0.7>	<0.2, 0.2, 0.5>	<0.1, 0.2, 0.5>	<0.1,0.3, 0.5>
Z3	<0.1, 0.5, 0.3>	<0.2, 0.2, 0.8>	<0.3, 0.1, 0.9>	<0.1, 0.1, 0.9>	<0.5,0.4, 0.1>
Demand	<0.1, 0.3, 0.7>	<0.2, 0.3, 0.5>	<0.1, 0.3, 0.5>	< 0.20415, 0.30416, 0.4>	

Table1 shows that reveals that outs of four sample respondents of cost, three respondents of zonals expectation level on the production offered by these promotors is Low, expectation level is Medium and remaining level have high level of expectation on the profits offered by manufacturing parts of automobile items

Automobiles company in coimbatore is a diversified automobile parts producing business in all over India. With annual cost over more than 10 dollars, automobiles related products are the largest portion of the company's business. To improve the company's sales and manufacturing performance, upper management concluded that it needed to acheieve four major objectives.

The work of customer service representative was to talk more than 10000 customer with accurate information about the availability of current and future inventory manufacturing order while considering requested delivery dates and maximum product age upon delivery. A second was to produce an efficient shift level schedule from each plant over a 30 day horizon. A third was to accurately determine whether a product was shipped on time and to check the item quantity to reach the customer on the requested date and time given the the avaibility of items and constraints on the zonals capacity.

4. Solution Method for Solving Fermatean Fuzzy Transportation Problem

Many specific strategies obtain possible in the composition for discovering a preliminary transparent feasible answer and an assumed answer for TP in contrasting fuzzy environs, viz., Intuitionistic fuzzy, Pythagorean fuzzy, etc. However, none of them have examined the transportation problem in Fermatean fuzzy environments, in which all the fields are FFS. We defined the transportation problem in Fermatean fuzzy surroundings by accepting the system's proponent set and using the perfectly perceived POM QM excel solver, as previously mentioned. The final steps of the recommended algorithm are during the time that followed.

Step 1:

Since k=1, 2, 3, determine the consecutive: $G_{kf} \langle \alpha_{p_i}, \beta_{p_i} \gamma_{p_i} \rangle$, i= 1,2,...,v,

 $G_{kf} \langle \alpha_{q_i}, \beta_{q_i} \gamma_{q_i} \rangle$, j= 1,2,...,w and

$$G_{kf} \langle \alpha_{k_{ij}}, \beta_{k_{ij}} \gamma_{k_{ij}} \rangle$$

Step 2 :

Since k=1, 2, 3, determine every consecutive:

 $G_{kf}\left(\sum_{i=1}^{\nu} \bigoplus \langle \alpha_{p_i}, \beta_{p_i}\gamma_{p_i} \rangle\right)$ and $G_{kf}\left(\sum_{j=1}^{w} \bigoplus \langle \alpha_{q_j}, \beta_{q_j}\gamma_{q_j} \rangle\right)$

Step 3:

If $G_{kf}(\sum_{i=1}^{\nu} \bigoplus \langle \alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} \rangle) = G_{kf}(\sum_{j=1}^{w} \bigoplus \langle \alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} \rangle)$, k=1,2,3 then the transportation problem

is balance

Step 4:

When $G_{kf}\left(\sum_{i=1}^{\nu} \bigoplus \langle \alpha_{p_i}, \beta_{p_i} \gamma_{p_i} \rangle\right) \neq G_{kf}\left(\sum_{j=1}^{w} \bigoplus \langle \alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} \rangle\right)$, k= 1,2,3 suggest a dummy element (G) or a dummy target (G) with zero transit cost and offer an extra opportunity to make it balance one and go to next step

Step 5:

Classified the balanced transportation problem as a linear programming problem and defined the use of the POM QM excel solver to discover an optimum solution. It is to be mentioned that to fix the

Pythagorean transportation problem by using the method, one can also replace the grade activity $G_{kf}(.)$ by $G_p(.)$

4.1. The proposed algorithm

- Step1:Check whether the given transportation problem is balanced or not. If not balance or by adding dummy row or column.
- **Step2:**Find the aggregate price and quantity index number for each row and each column. Then identify the lowest values corresponding for each row and each column.
- **Step 3:**Allocate the minimum aggregate price and quantity index numbers in the supply or demand at the place of minimum value of the corresponding row or column of the table.
- **Step 4:**Repeat the step 2 and 3 until all the demands are satisfied and all the supplies are Exhausted.
- **Step 5:**Total Fermatean fuzzy transportation problem of minimum cost equal to sum of the product of the cost and its corresponding allocated values of supply or demand
- Table 2: Grade activity using modified Leading indicator (LI) of cost, supply and demand model I

Item	Ι	Π	III	IV	Supply	LI-1	LI -2	LI -3	LI -4
Z_1	0.3250	0.1390	0.4050	0.3885	0.5580 <u>0.3355</u> 0.2225	0.3144	0.3108	0.3108	0.3960
Z ₂	0.2770	0.3320	0.4375	0.4340	0.5485 <u>0.3390</u> 0.2095	0.3701	0.4012	_	_
Z ₃	0.4245	0.2440	0.1485	0.1355 0.2553	0.4275 <u>0.2553</u> 0.1722	0.2381	0.176	0.176	0.142
Z_4	0	0	0	0	0.3763	0	-	_	-
Demand	0.3390	0.5450 <u>0.2095</u> 0.3355	0.5485 <u>0.1722</u> 0.3763	0.4778 <u>0.2225</u> 0.2553					
LI -1	0.2567	0.1787	0.2477	0.2395					
LI -2	-	0.1787	0.2477	0.2395					
LI -3	-	0.1276	0.1845	0.1746					
LI -4	-	-	0.1845	0.1746					

Table 2 shows that the initial basic feasible solution by using our proposal method. Since the transportation problem is a balanced. So solution is optimal and unique. The optimal solution is given by X_{12} is 0.3355, X_{14} is 0.2225, X_{21} is 0.3390, X_{12} is 0.3355, X_{21} is 0.2095, X_{33} is 0.1722, X_{34} is 0.2553, X_{43} is 0.3783. Then minimum transportation Cost is 0.3567. To meet these challenges,we developed an integrated sytem of linear programming models based on the three category formulations to dynamically schedule its automobile product operations at three zonals in real time as it receives orders. The total audited benefits realized in the first year of operation of this

system were 35.674 millions, including 35 million due to optimizing the product mix. Other benefits include a reduction in order, a reduction in price based on discounting and better on time delivery.

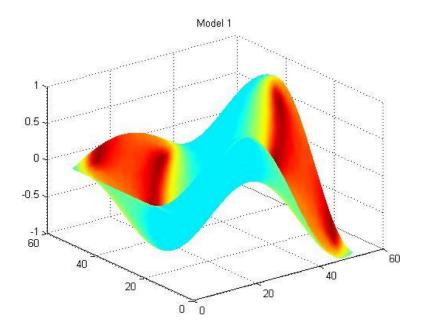


Figure 2. Model I

Figure 2 shows that total audited benefits realized in the first year of operation of this system were 35.674 millions, including 35 million due to optimizing the product mix. Other benefits include a reduction in order, a reduction in price based on discounting and better on time delivery.

Table -3: Grade activity	using modified L	eading indicator (LI) of cost,	supply and dema	and model - II

Item	I	II	ш	IV	Supply	LI-1	LI-2	LI-3	LI-4	LI-5
	0.217	0.093	0.379	0.259	0.2896 0.2880					
Z_1	-	0.2880	-	0.0016	0.0016	0.2372	0.2439	0.2439	0.3194	-
	0.178	0.302	0.294	0.240	0.2833 0.2106					
Z_2	0.2106	-	_	0.0727	0.0727	0.2539	0.2791	-	-	-
	0.261	0.165	0.082	0.090	0.271	0.1498	0.1126	0.1126	0.0863	0.0863
Z_3	-	_	0.271							
	0	0	0	0	0.2861					
Z_4			0.0521	0.235	<u>0.054</u> 0.2350	-	-	-	-	
Demand	0.2106	0.2880	0.3231 <u>0.2710</u> 0.0521	0.3083 <u>0.0727</u> 0.2356						
				0.0016 0.2340						
LI-1	0.1641	0.1401	0.1889	0.1476						
LI-2	-	0.1401	0.1889	0.1476	1					
LI-3	-	0.0861	0.1538	0.1168	1					
LI-4	-	-	0.1538	0.1168	1					
LI-5	-	-	0.042	0.0584	1					

The fermatean fuzzy TP approach used this method to find the optimal solutions X_{12} is 0.2880, X_{14} is 0.0016, X_{21} is 0.2106, X_{24} is 0.0727, X_{33} is 0.271, X_{42} is 0.0521, X_{44} is 0.235 with and the minimum transportation Cost 0.10304.

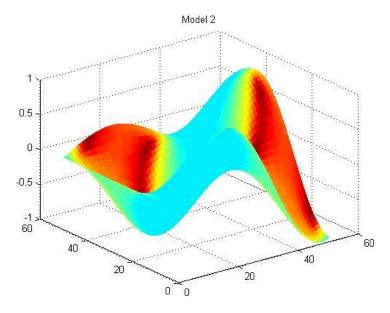


Figure 3. Model II

Figure 3 shows that the solution indicates that the automobiles company should produce products I,II,III,IV at the rate of zonals respectively, with a resulting total profit of 10.39 millions per annum.

Table -4: Grade activity using modified Leading indicator (LI) of cost, supply and demand

Item	I	II	III	IV	Supply	LI-1	LI-2	LI-3
Z_1	0.260	0.1112	0.1620	0.1165	0.1116	0.1624	0.1624	0.1624
	0.155	0.1992	0.2188	0.2604	0.0548	0.2083	0.2083	-
Z ₂	0.0548	-	-	-				
Z_3	0.297	0.6048	0.1038 0.0499	0.1218	0.0499	0.2819	-	-
Z.4	0 0.116	0 0.1012	0.005	0.0175	0.2397 <u>0.1160</u> 0.1237 <u>0.1012</u> 0.0225 <u>0.0050</u> 0.0175	0	0	0
Demand	0.1708 <u>0.0548</u> 0.116	0.2128 0.1116 0.1012	0.0549 <u>0.0499</u> 0.005	0.0175				
LI-1	0.1780	0.2288	0.1211	0.1245				
LI-2	0.1383	0.1034	0.1269	0.1256				
LI-3	0.13	0.0556	0.081	0.0582				

Table 4 shows that the initial basic feasible solution by using our proposal method, since the transportation problem is a balanced. Then solution is optimal and unique. So optimal solution is given by X_{12} is 0.1116, X_{21} is 0.0548, X_{33} is .0499. Then minimum transportation Cost 0.026.

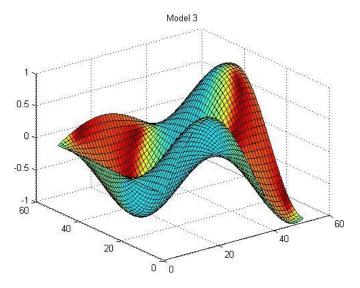


Figure 4. Model III

Figure 4 shows that the most favourable value is the largest value if the objective function is to be maximized, whereas it is the smallest value if the objective function is to be minimized. This solution indicates that the automobiles company should produce products I,II,III,IV at the rate of 4 zonals respectively, with a resulting total profit of 2.6 millions per annum.

5. Development and analysis by an arithmetical illustration

Now the present part, we suggested explanation technique by an applicable consider a Fermatean fuzzy transportation problem, point everyone every domain go on liable current Table 1, current that illuminate, surplus $\langle \alpha_{p_i}, \beta_{p_i}, \gamma_{p_i}, \rangle$, i=1,2,3,requirement $\langle \alpha_{q_i}, \beta_{q_i}, \gamma_{q_i}, \rangle$, j=1,2,3,4 along with $\langle \alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}}, \rangle$, i=1,2,3, j= 1,2,3,4 along with contemplated on the point of FFNs in this direction Z₁, Z₂, Z₃ go on three element and I,II,III,IV go on four channels. Through find out the FFTP, they obtain resolved every subsequent LPP (3) position everyone every information go on liable now Table 1.

5.1. Model 3

 $\begin{array}{ll} \text{Minimize} & G_{Df} < \alpha_{z_0}, \beta_{z_0}, \gamma_{z_0} > = \sum_{i=1}^{3} \sum_{j=1}^{4} G_{Df}(<\alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}}, >) \odot z_{ij} \end{array} \tag{3} \\ \begin{array}{ll} \text{Subject to} \\ \sum_{j=1}^{4} z_{ij} \leq G_{Df}(<\alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} >), \text{ i=1,2,3} \\ \sum_{i=1}^{3} z_{ij} \leq G_{Df}(<\alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} >), \text{ j=1,2,3,4} \\ \end{array} \\ \text{Where } 0 \leq (\alpha_{z_0})^3 + (\alpha_{z_0})^3 + (\alpha_{z_0})^3 \leq 1 \end{array}$

$$0 \le (\alpha_{z_o})^3 + (\alpha_{z_o})^3 + (\alpha_{z_o})^3 \le 1, \ 0 \le (\alpha_{z_o})^3 + (\alpha_{z_o})^3 + (\alpha_{z_o})^3 \le 1$$
$$z_{ij} \ge 0, 0 \le (\alpha_{k_{ij}})^3 + (\alpha_{k_{ij}})^3 + (\alpha_{k_{ij}})^3 \le 1, \ i=1,2,3, \ j=1,2,3,4$$

Exacting category 1, category 2, furthermore category 3, and grade function further more accept accomplished model 3 particularly model 4, furthermore model 5 and that go on the point of pursue.

5.2.Model 4

Minimize
$$G_{1f} < \alpha_{z_0}, \beta_{z_0}, \gamma_{z_0}, > = \sum_{i=1}^3 \sum_{j=1}^4 G_{1f}(<\alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}}, >) \odot z_{ij}$$
 (4)

Subject to

$$\begin{split} & \sum_{j=1}^{4} z_{ij} \leq G_{1f}(<\alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} >), \text{ i=1,2,3} \\ & \sum_{i=1}^{3} z_{ij} \leq G_{1f}(<\alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} >), \text{ j=1,2,3,4} \\ & z_{ij} \geq 0, \ 0 \leq \left(\alpha_{k_{ij}}\right)^3 + \left(\alpha_{k_{ij}}\right)^3 + \left(\alpha_{k_{ij}}\right)^3 \leq 1, \text{ i=1,2,3, j=1,2,3,4} \end{split}$$

The Grade character comparable toward transport expenditure surplus further more requirements about model 4 go on liable current table 6.

5.3.Model 5

Minimize

$$G_{2f} < \alpha_{z_0}, \beta_{z_0}, \gamma_{z_0}, > = \sum_{i=1}^{3} \sum_{j=1}^{4} G_{2f}(<\alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}}, >) \odot z_{ij}$$
(5)

Subject to

$$\begin{split} & \sum_{j=1}^{4} z_{ij} \leq G_{2f}(<\alpha_{p_i}, \beta_{p_i}, \gamma_{p_i} >), \text{ i=1,2,3} \\ & \sum_{i=1}^{3} z_{ij} \leq G_{2f}(<\alpha_{q_j}, \beta_{q_j}, \gamma_{q_j} >), \text{ j=1,2,3,4} \\ & z_{ij} \geq 0, \text{i=1,2,3, j=1,2,3,4} \end{split}$$

Every Grade expense comparable toward transport expenditure, surplus furthermore requirement about model 5 go on liable current table 7.

5.4. Model 6

Minimize
$$G_{3f} < \alpha_{z_0}, \beta_{z_0}, \gamma_{z_0}, \gamma = \sum_{i=1}^3 \sum_{j=1}^4 G_{3f}(<\alpha_{k_{ij}}, \beta_{k_{ij}}, \gamma_{k_{ij}}, >) \odot z_{ij}$$
 (6)

Subject to

$$\begin{split} & \sum_{j=1}^{4} z_{ij} \leq G_{3f}(<\alpha_{p_i}, \beta_{p_i}, \gamma_{p_i}>), \text{ i=1,2,3} \\ & \sum_{i=1}^{3} z_{ij} \leq G_{3f}(<\alpha_{q_j}, \beta_{q_j}, \gamma_{q_j}>), \text{ j=1,2,3,4}, z_{ij} \geq 0, \text{ i=1,2,3, j=1,2,3,4} \end{split}$$

Table -5:Grade activity of Model (3)

MODEL 3	Ι	II	III	IV	Supply
Z1	0.3250	0.1390	0.4050	0.3885	0.5580
Z ₂	0.2770	0.3320	0.4375	0.4340	0.5485
Z ₃	0.4245	0.2440	0.1485	0.1355	0.5300
Demand	0.3390	0.5450	0.5485	0.4778	1.6365≠1.9103

Table 5 shows that stage now was set to develop the general framework of the automobile company's plan for model 1. This plan specifies which types of grade activity $G_{1P}(P)$ will be used and at what

fractions of their (model 3), crisp capacities crisp demand, (2) the crisp cost value. Because of the combinatorial nature of the problem of finding a plan that satisfies the requirements with the smallest possible cost, and this was formed to solve the problem. The adopted a fuzzy fermatean TP approach, formulating the crisp value

MODEL 4	Ι	II	III	IV	Supply
Z_1	0.217	0.093	0.3795	0.2593	0.2896
Z ₂	0.1783	0.3023	0.2943	0.2407	0.2833
Z ₃	0.2613	0.1653	0.082	0.0906	0.271
Demand	0.2106	0.288	0.3231	0.3083	0.8412≠1.13

Table -6: Grade activity corresponding to transportation cost, supply and demand of Model (4)

Table 6 shows that within the restrictions specified in Tables, automobiles company management wants to determine the *amount* of each product grade to produce *and* the exact *mix* of the materials to be used for each grade activity $G_{2P}(P)$. The objective is to maximize the net weekly profit (total sales income *minus* total amalgamation cost).

Table -7: Grade activity of Model (5)

MODEL 5	Ι	II	III	IV	Supply
Z ₁	0.2600	0.1112	0.1620	0.1165	0.1116
Z ₂	0.1549	0.1992	0.2188	0.2604	0.0548
Z ₃	0.2972	0.6048	0.1038	0.1218	0.0499
Demand	0.1708	0.2128	0.0549	0.0175	0.7095≠0.456

Table 7 shows that grade activity corresponding to transportation cost, supply and demand of model (3) Commercial conclusion of model (2), model (3), and model (4), model (2), model (3) along with model (4) have been resolved applying excel solver along with arithmetical impact have been granted with it Table (5).

Table -8 :

Grade activity of FFTP of Model(6)

Problem	Grade activity	Optimal solutions	Minimum cost
Model (3)	$\frac{1}{2}\left(1+\alpha_P^3-\beta_P^3-\gamma_P^3\right)$	$x_{12} = 0.55, \ X_{21} = 0.34.$	$R_{1P} (\mathbf{P}) \langle \alpha_{P(x)}, \beta_{P(x)}, \gamma_{P(x)} \rangle = 0.3283$
		$x_{22} = 0.17, \ x_{24} = 0.04$	
		$x_{34} = 0.43, x_{34} = 0.38, X_{44} = 0$	
Model (4)	$\frac{1}{3}\left(1+2\alpha_P^3-\beta_P^3-\gamma_P^3\right)$	$x_{12} = 0.29, x_{21} = 0.21,$ $x_{24} = 0.07, x_{33} = 0.27$	$R_{2P}(\mathbf{P})\langle \alpha_{P(x)}, \beta_{P(x)}, \gamma_{P(x)} \rangle = 0.1023$
Model (5)	$\frac{1}{2}\left(1+\alpha_P^3-\beta_P^3-\gamma_P^3\right) \alpha_P-\alpha_P^3 $	$ \beta_P - \gamma_P x_{12} = 0.12, \ x_{21} = 0.05,$ $x_{33} = 0.05, \ x_{33} = 0$	$R_{3P}(\mathbf{P})\langle \alpha_{P(x)}, \beta_{P(x)}, \gamma_{P(x)}\rangle = 0.0257$

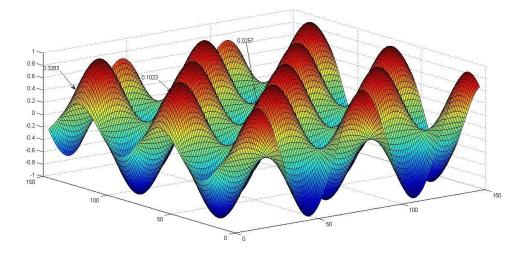


Figure 5. Grade activity of result

Figure 5 shows that the process of never organization have resolved Transportation Problem in consideration of Fermatean Fuzzy magnitude, hence never comparability have been formed since the equal [19]. Such form (3) grade activity is exceptional than form (2) grade activity is exceptional form (1) grade activity taken away table (5) it act been that, the smallest cost captured they can request that their planned technique is a modern approach to accoutrement the doubtfulness. Fermatean fuzzy circumstances[12]. Continuing to apply this procedure to the automobiles company problem yields the complete set of transportation shown in above table 5. Since all the *optimal values* are nonnegative in the fifth table, the optimality test identifies the set of allocations in this table as being optimal, which concludes the algorithm. It would be good practice for you to derive the values grade activity given in the second, third, and fourth table. Table 8 shows that doing this by working directly on the table. Also check out the chain reactions in the second and third table, which are somewhat more complicated.

5.5. Comparative Study

We consider three types of models using production in automobiles and converting into fuzzy numbers. We use the grade and veracity activity to convert it into a crisp value. We solve that crisp value with the fermatean fuzzy transportation problem concept and by the fermatean fuzzy transportation problem concept where we analyze the optimal solutions bellow table 9

Method	Fermatean fuzzy transportation Problem using grade activity	Fermatean fuzzy transportation Problem using veracity activity
Minimize total Manufacturing cost Model 3	35.67 %	32.83%
Minimize total Manufacturing cost Model 4	10.3 %	10.23%
Minimize total Manufacturing cost Model 5	2.57%	2.63%

Table -9: Compartive study

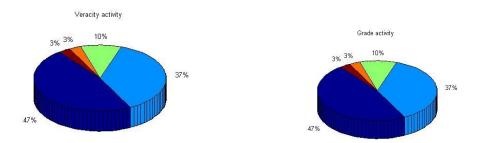


Figure 6. Comparative study of result

Figure 6 shows that we compare the minimum cost model 3 and maximum profits in model 1 from fermatean fuzzy transportation problem

6. Conclusion

In this study, we introduce a technique for addressing fuzzy TP at their optimal level. Additionally, we have spoken about how the parameters for the transportation problem are FFS. In making decisions, the conception of the transportation problem is crucial. The distinction between a FFS and a PFS was also demonstrated. When, at a particular time, the decision-makers lack sufficient information regarding surplus and requirement and transit cost, etc. The transportation problem with various parameters can benefit greatly from the FFS parameters. Additionally, the FFS arithmetic operation is used to find the best solution. FFSTP is a powerful technique for dealing with the problem of allocating limited resources among competing activities as well as other problems having a similar mathematical formulation. It has become a standard tool of great importance for numerous business and industrial organization. Furthermore, almost any social organization is concerned with allocating resources in some context, and there is growing recognition of the extremely wide applicability of this technique.

The linear Programming problem implicating targets and transformed into a transportation problem with Fermatean Fuzzy Numbers(FFNs). These uncertain pieces of information were processed in determination making using Fermatean Fuzzy Sets. An algorithm has been proposed for proving our proposed method concerning Fermatean Fuzzy Parameters(FFPs) and also the optimum value has been completed by using software TP in Fermatean Fuzzy Numbers(FFNs). Using the fermatean fuzzy transportation approach, a model was developed, which provided different types models between automobiles company products. The single-objective model focused only on minimizing the products while the different kinds of model provided a more balanced solution that considered manufacturing supply and demand products. The comparison between these models showed that the different kinds models approach provided the best trade-off between supply and demand of cost

optimized value. While the single-objective approach resulted in the lowest optimized value, but also the lowest cost. Overall, the fermatean fuzzy transportation problem approach provided a useful tool for decision-makers to optimize production while minimizing cost in automobile company. The results showed that different models have different products supply and demand, and the optimal solution depends on the specific goals and priorities of the decision-maker. By considering different objectives and using fuzzy logic, the fermatean fuzzy transportation problem approach can help to find more robust and balanced solutions that take into account the complex and uncertain nature of systems. In future studies, the approach proposed in this paper may be taken into consideration to solve multi objective programming problem packages to other fuzzy methods such as q-rung orthopair fuzzy set(q-ROFSs) can be apply on the Fermatean fuzzy transportation problem. The Fermatean fuzzy transportation problems in medical diagnosis under Fermatean fuzzy soft data using grade activity will be discussed.

7. References

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